Journal of Applied Mathematics and Mechanics 81 (2017) 431–441

Contents lists available at ScienceDirect

Journal of Applied Mathematics and Mechanics

journal homepage: www.elsevier.com/locate/jappmathmech

Steady rotations of a satellite with internal elastic and dissipative forces☆

N.I. Amelkin*, V.V. Kholoshchak

Moscow Institute of Physics and Technology, Dolgoprudnyi, Russia

ARTICLE INFO

Article history:
Received 19 January 2017
Available online 7 May 2018

Keywords:
rigid body
inertia tensor in a circular orbit
orbital plane
dissipative torque
gravitational torque

ABSTRACT

Lavrent'ev's model (a satellite is simulated by a rigid shell with a spherical damper) is used to study the effect of internal forces on the motion of a satellite in a central gravitational field assuming that both dissipative and elastic internal forces arise with relative displacements of the damper. All the steady rotations are determined within the framework of this model for a dynamically symmetric satellite in a circular orbit and their stability is investigated as a function of the values of the damping and stiffness coefficients.

1. Statement of the problem. Equations of motion

The investigation of the steady motions of mechanical systems with internal dissipation is of interest in relation to the fact that, in the majority of cases, the steady motions are the unique limit motions of such systems. The steady motions of a dynamically symmetric satellite with a spherical damper in a central gravitational field when there are only internal dissipative forces have been studied earlier [1] within the framework of a circular restricted problem. A similar problem for the case when external elastic forces act in addition to dissipative forces is considered below.

A satellite is considered that consists of a supporting rigid body (shell) and a uniform spherical body (damper) carried by the shell with the centre of the uniform body (damper) fixed with respect to the shell (Lavrent'ev's model [2,3]). The vector of the absolute angular velocity of the shell is denoted by $\boldsymbol{\omega}$ and the absolute angular velocity of the damper is denoted by Ω. The angular momentum of the satellite with respect to its centre of mass is described in the form

$$\mathbf{K} = \mathbf{K}_1 + \mathbf{K}_2; \qquad \mathbf{K}_1 = \mathbf{J}^*\omega, \qquad \mathbf{K}_2 = I\,\Omega; \quad \mathbf{J}^* = \mathbf{J} - I\,\mathbf{E} \tag{1.1}$$

where $\mathbf{K_1}$ is the angular momentum of the *auxiliary* body formed by the shell and a point mass equal to the mass of the damper located at the centre of the damper, $\mathbf{K_2}$ is the angular momentum of the damper, $\mathbf{J}^*$ is the inertia tensor of the auxiliary body, $\boldsymbol{I}$ is the moment of inertia of the damper with respect to its central axis, $\mathbf{J}$ is the central inertia tensor of the whole satellite with principal moments of inertia A, B, C, and $\mathbf{E}$ is a unit matrix.

We shall assume that, with relative displacements of the damper, a dissipative torque $\mathbf{M}_d$ and an elastic force torque $\mathbf{M}_e$, defined by the formulae

$$\mathbf{M}_d = -\tilde{\mu}\,I(\boldsymbol{\Omega} - \boldsymbol{\omega}),\ \mathbf{M}_e = -\tilde{k}\,I\,\boldsymbol{\theta};\ \ \boldsymbol{\theta} = 2\xi\,\mathrm{tg}\frac{\phi}{2} \tag{1.2}$$

act on it.

☆ *Prikl. Mat. Mekh.*, Vol. 81, No. 6, pp. 627-641, 2017.
* Corresponding author.
E-mail addresses: namelkin@mail.ru (N.I. Amelkin), khoviktoriya@yandex.ru (V.V. Kholoshchak).

https://doi.org/10.1016/j.jappmathmech.2018.03.011
0021-8928/

Here, $\tilde{\mu}$ and $\tilde{k}$ are damping and stiffness coefficients, θ is the vector of the finite rotation of the damper with respect to the shell, ξ is the unit vector of the axis of the finite rotation and ϕ is the angle of finite rotation. The potential energy of the elastic forces is defined by the expression

$$\Pi_e = 2\tilde{k}I\ln\left(1+\frac{\theta^2}{4}\right) \tag{1.3}$$

We next assume that the rotational motion of the satellite and the motion of its centre of mass are mutually independent and that the centre of mass moves in a circular orbit. In this case, the gravitational torque acting on the satellite has the form [4]

$$\mathbf{M}_g = 3\omega_0^2\,\mathbf{r}\times\mathbf{Jr} \tag{1.4}$$

where ω_0 is the angular velocity vector of the orbital basis directed along the normal $\mathbf{n}$ to the orbital plane and $\mathbf{r}=\mathbf{R}/R$, where $\mathbf{R}$ is the radius vector joining the centre of attraction to the centre of mass of the satellite.

Using the dimensionless time $\tau=\omega_0 t$ and the dimensionless variables

$$\mathbf{U} = \omega/\omega_0, \quad \mathbf{V} = \Omega/\omega_0 \tag{1.5}$$

the equations for the rotational motion of the satellite can be written in the form

$$\frac{d\mathbf{L}_1}{d\tau} = (\mathbf{J}-I\mathbf{E})\mathbf{U}' + \mathbf{U}\times\mathbf{JU} = 3\mathbf{r}\times\mathbf{Jr} + \mu I(\mathbf{V}-\mathbf{U}) + kI\theta \tag{1.6}$$

$$\frac{d\mathbf{L}_2}{d\tau} = I(\mathbf{V}'+\mathbf{U}\times\mathbf{V}) = -I(\mu(\mathbf{V}-\mathbf{U})+k\theta) \tag{1.7}$$

$$2\Lambda' = \Lambda\circ\mathbf{U} \tag{1.8}$$

$$\theta' = (\mathbf{V}-\mathbf{U}) + \frac{1}{2}(\mathbf{V}-\mathbf{U})\times\theta + \frac{1}{4}\theta(\theta\cdot(\mathbf{V}-\mathbf{U})) \tag{1.9}$$

In these equations, derivatives with respect to the dimensionless time with reference to the Koenig basis are denoted by the symbol $\frac{d}{d\tau}$ and derivatives in the basis $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ associated with the shell are denoted by a prime. The left-hand sides of Eqs. (1.6) and (1.7) are derivatives of the reduced angular momentum of the auxiliary body and the reduced angular momentum of the damper

$$\mathbf{L}_1 = \mathbf{K}_1/\omega_0 = (\mathbf{J}-I\mathbf{E})\mathbf{U}, \quad \mathbf{L}_2 = \mathbf{K}_2/\omega_0 = I\mathbf{V}$$

and the right-hand sides are the reduced acting torques. The dimensionless damping coefficient and stiffness coefficient:

$$\mu = \tilde{\mu}/\omega_0, \quad k = \tilde{k}/\omega_0^2$$

are denoted by μ and k.

Kinematic equations (1.8) describe the rotational motion of the auxiliary body, where Λ is the unit norm quaternion specifying the position of the basis of the principal axes of inertia $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ of the satellite with respect to the Koenig basis. Equations (1.9) express the relation between the finite rotation velocity vector of the damper and the relative angular velocity of the damper.[5]

The components of the vector $\mathbf{r}$ in the $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ basis are defined by the formulae [1]

$$\mathbf{r} = \bar{\mathbf{N}}\circ\mathbf{e}_1\circ\mathbf{N}, \quad \mathbf{N} = \bar{\mathbf{M}}\circ\Lambda, \quad \mathbf{M} = \left(\cos\frac{\tau}{2}, 0, 0, \sin\frac{\tau}{2}\right)$$

The system of equations (1.6)–(1.9) is non-autonomous since the components of the vector $\mathbf{r}$ in the $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ basis are explicitly dependent on time. To obtain an autonomous system of equations, it is necessary to use phase variables which determine the state of the satellite with respect to the orbital basis formed by the vectors $\mathbf{n}$, $\mathbf{r}$ and $\boldsymbol{\tau}=\mathbf{n}\times\mathbf{r}$.

For a dynamically symmetric satellite ($A=B\neq C$), the reduced angular momentum of the auxiliary body and the reduced gravitational torque are expressed by the formulae

$$\mathbf{L}_1 = (A-I)\mathbf{U} + (C-A)(\mathbf{U}\cdot\mathbf{e})\mathbf{e}, \quad m_g = 3\mathbf{r}\times\mathbf{Jr} = 3(C-A)(\mathbf{r}\cdot\mathbf{e})\mathbf{r}\times\mathbf{e} \tag{1.10}$$

where $\mathbf{e}$ is the unit vector of the axis of dynamic symmetry. The angular velocities of the shell and the damper in the orbital basis are denoted by $\mathbf{u}$ and $\mathbf{v}$ respectively and derivatives with respect to the dimensionless times, calculated in the orbital basis, are denoted by a dot. Taking account of the fact that the dimensionless angular velocity of the orbital basis is equal to $\mathbf{n}$ and using the equality

$$\mathbf{u} = \mathbf{U}-\mathbf{n}, \quad \mathbf{v} = \mathbf{V}-\mathbf{n}, \quad \frac{d\mathbf{U}}{d\tau} = \mathbf{n}\times\mathbf{u}+\dot{\mathbf{u}}, \quad \frac{d\mathbf{V}}{d\tau} = \mathbf{n}\times\mathbf{v}+\dot{\mathbf{v}}, \quad \dot{\mathbf{e}} = \mathbf{u}\times\mathbf{e}, \quad \dot{\theta} = \mathbf{u}\times\theta+\theta' \tag{1.11}$$

we obtain the following autonomous system of equations of motion of a dynamically symmetric satellite:

$$\dot{\mathbf{e}} = \mathbf{u}\times\mathbf{e} \tag{1.12}$$

$$\begin{aligned}&(A-I)(\mathbf{n}\times\mathbf{u}+\dot{\mathbf{u}}) + (C-A)[((\mathbf{n}\times\mathbf{u}+\dot{\mathbf{u}})\cdot\mathbf{e})\mathbf{e} + ((\mathbf{n}+\mathbf{u})\cdot\mathbf{e})(\mathbf{n}+\mathbf{u})\times\mathbf{e}] =\\ &= 3(C-A)(\mathbf{r}\cdot\mathbf{e})(\mathbf{r}\times\mathbf{e}) + I[\mu(\mathbf{v}-\mathbf{u})+k\theta]\end{aligned} \tag{1.13}$$

$$I(\mathbf{n}\times\mathbf{v}+\dot{\mathbf{v}}) = -I[\mu(\mathbf{v}-\mathbf{u})+k\theta] \tag{1.14}$$