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The stability of the motion of a non-axisymmetric body in a resistive medium☆

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ABSTRACT

A mathematical model of the plane motion of a non-axisymmetric body in a resistive medium is constructed using the local interaction method. Criteria for the stability of rectilinear motion are obtained in a general form in the case of a frozen axial velocity. The stability of the motion of a regular triangular pyramid is investigated in detail when a constant friction and pressure, specified using an empirical Poncelet formula in the form of a sum of inertial and strength terms, acts on its lateral surface. The stabilities of a pyramid and a cone are compared. The effect of deceleration on the stability of the rectilinear motion is considered.

As a rule, it is only possible, due to the complex nature of the interaction forces, to solve problems on penetration, the optimization of the shape of a body and the stability of its motion in an exact formulation using difference methods. However, on account of the large number of parameters and governing functions, such calculations often turn out to be of little use in revealing the general regularities of the penetration process. Approximate approaches are therefore often used in studying the effect of the parameters of the medium and the body on the trajectory of the motion. One such approach is the local interaction method that had already been proposed for the first time by Newton for determining the resistance to the motion of bodies in gases and liquids. At the present time, this method is used to solve hypersonic flow problems, problems of motion in a rarefied gas and the penetration of a body into soils and metals.[1–3]

The stability of the motion of a solid body has been studied for the case of a frozen velocity of the motion of the centre of mass and the absence of rotation about the axis of symmetry.[4–10] Stability in the case of the plane motion of a thin cone and a pyramidal body with two or four symmetry planes has been investigated. [4,10] Stability criteria for a thin body of revolution taking account of flow separation from its lateral surface have been obtained for plane and three-dimensional motions.[5–8] Subsequently, the requirement that the body was thin was successfully dropped and stability criteria for an axisymmetric body of finite thickness were successfully found,[9] and it was also shown that, when there is no rotation about the symmetry axis, the criteria for the stability of a rectilinear motion in the three-dimensional case are identical [7,9] to the stability criteria obtained when a plane motion is considered.

The stability of the rectilinear motion of a body in a resistive medium in the case of a plane motion is studied below on the basis of the local interaction method. A mathematical model is constructed and stability criteria are obtained in a general form for a non-axisymmetric body which makes the mathematical model considerably more complicated. Moreover, unlike in earlier investigations,[4–8,10] the motion of a body of arbitrary thickness is considered and the problem is solved for a more general specification of the contact pressure and the shear stresses which extends the range of possible applications.

1. Physical description of the problem and hypotheses

A rigid body with a length L_m, radius R_b and mass m moves by inertia in an unbounded medium. The cylindrical system of coordinates R, φ, $L (0 \le \varphi \le 2\pi)$ and the local rectangular system of coordinates associated with the centre of mass

$$x = (L_c - L)/L_m, \quad y = R\cos\varphi/L_m, \quad z = R\sin\varphi/L_m$$

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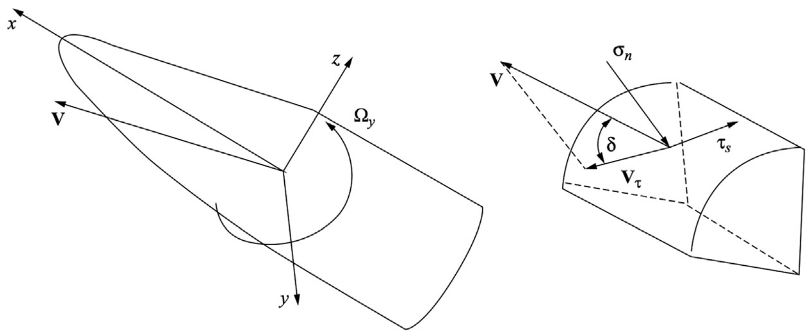

Fig. 1.

are rigidly bound with the body. The body is assumed to be symmetric with respect to the *xz* plane, has a moment of inertia I_y with reference to the *y* axis and its surface is given by the equation

$$R_b = R_b(L, \varphi), \quad R_b(L, \pi/2 - \varphi) = R_b(L, \pi/2 + \varphi)$$

where *L* is a distance from the tip along the body axis. It is assumed that motion of the body is a plane motion and it is determined by the velocity vector $\mathbf{v} = \{\upsilon_x, 0, \upsilon_z\}$ of the centre of mass located at $L = L_c$ and by the angular velocity of rotation about the centre of mass $\Omega = \{0, \Omega_y, 0\}$ which are dependent on the time *t* (Fig. 1). It is assumed that there is a continuous flow past the lateral surface of the body but separation of the flow from the rear edge of the body is possible (when $L = L_m$).

We introduce the new dimensionless quantities

$$r = \frac{R_b}{L_m}, \quad l = \frac{L}{L_m}, \quad l_c = \frac{L_c}{L_m}, \quad \gamma = \frac{dr}{dl}, \quad \beta = \frac{1}{r}\frac{dr}{d\varphi}, \quad \eta_z = \frac{\upsilon_z}{\upsilon_x}, \quad \omega_y = \frac{L_m \Omega_y}{\upsilon_x}$$
$$ds = \frac{\upsilon_x dt}{L_m}, \quad s(t = 0) = 0$$

The expressions for the outward normal and the projections of the velocity vector **V** of an arbitrary point on the surface of the body onto the normal and onto the tangent plane in the system of coordinates *xyz* are defined by the formulae

$$\mathbf{R} = L_m\{x, y, z\}, \quad \mathbf{V} = \mathbf{v} + [\Omega \times \mathbf{R}] = v_x\{1 + \omega_y z, 0, -a_3\}$$
$$V = |\mathbf{V}| = \Theta \upsilon_x, \quad V_n = (\mathbf{n} \cdot \mathbf{V}) = \Theta v_x \sin\delta, \quad \mathbf{V}_\tau = \mathbf{V} - \mathbf{n} V_n, \quad \mathbf{n}_\tau = -\mathbf{V}_\tau / |\mathbf{V}_\tau|$$
$$a_3 = \omega_y x - \eta_z, \quad \Theta = \sqrt{(1 + \omega_y z)^2 + a_3^2}$$
$$\mathbf{n} = \{\gamma, \cos\varphi + \beta\sin\varphi, \sin\varphi - \beta\cos\varphi\}\zeta^{-1}, \quad \sin\delta = (\gamma - A_2\cos\varphi - A_3\sin\varphi)(\Theta\zeta)^{-1}$$
$$\zeta = \sqrt{1 + \gamma^2 + \beta^2}, \quad A_2 = -a_3\beta, \quad A_3 = a_3 - \gamma\omega_y r \tag{1.1}$$

Here, δ is the angle of attack (the angle between the tangent plane and a certain point on the surface of the body and the velocity vector of this point). The angle δ is negative in the “dead air” ($V_n < 0$) region.

We represent the stress vector on the surface of the body in the form

$$\Sigma = \tau_s \mathbf{n}_\tau - \sigma_n \mathbf{n}$$

where τ_s is the shear stress, $\mathbf{n}_\tau$ is a unit tangent vector in the direction of slip and σ_n is the contact pressure. The shear stress and the contact pressure are represented in the form of functions of the dimensionless quantities *l*, φ, κ, δ and Θ:

$$\tau_s = \tau_d \tau, \quad \sigma_n = \frac{m v_x^2}{L_m^3}\sigma; \quad \tau = \tau(l, \varphi, \kappa, \delta, \Theta), \quad \sigma = \sigma(l, \varphi, \kappa, \delta, \Theta), \quad \kappa = \frac{L_m^3 \tau_d}{m v_x^2} \tag{1.2}$$

Here, τ and σ are dimensionless differentiable functions and τ_d is a constant with the dimensions of Pascal (Pa). In the case when $\tau_s = \text{const} \neq 0$, it is possible to put $\tau_d = \tau_s$. The relation between τ and σ and δ and κ is introduced to take account of the effect of the local angle of attack and the axial velocity of the motion of the centre of mass of the body υ_x on the contact pressure and the shear stresses. The dependence on the quantity Θ enables us to change from the axial velocity υ_x to the velocity of the motion of an elementary small area *V* and the dependence on *l* and φ enables us to take account of the reduction in the stresses close to the rear edge of the body and the effect of the curvature of the lateral surface on the stresses. In solving a specific problem, the relation between the contact stresses and these quantities is determined by the properties of the medium into which the body penetrates and can be obtained from a calculation of the motion of the body in an exact formulation using difference methods.

Although the hypothesis of locality cannot correspond to the physical picture of the flow, representation of the normal stresses in the form of (1.2) enables us to describe the normal stress distribution on the surface of a body in different media with an acceptable accuracy.[2] At the same time, the approximation that the shear stress vector acts in the direction of slip is a fairly rough approximation. The assumption that the velocity field on the surface of a body is the same as for a steady motion in an incompressible fluid [11] is more frequently used in the case of the high-velocity motion of a body in an elastoplastic medium. Nevertheless, this representation enables us to estimate the effect of shear stresses on the stability and trajectory of the motion.