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On methods for increasing the margin of stability of motion of optimum bodies☆

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ABSTRACT

The possibilities of increasing the margin of stability of motion of optimum bodies having minimum drag or maximum penetration depth during high-velocity motion in a dense medium are investigated. It is assumed that the stresses generated by the medium acting on a surface element of the body are described within the framework of the local interaction model by binomial formulae quadratic in the velocity. A study has been performed for the case when the body shape is taken to be prescribed and when it is possible to vary it without departing from the class of optimum bodies. It is shown that for a fixed shape the simplest ways to increase the margin of stability of motion of the body are to decrease its mass or to move the centre of mass of the body closer to its vertex. It is possible to increase the margin of stability of motion of the body without decreasing its mass and without breaching the homogeneity of the body if it is equipped with fins. A method has been developed for constructing homogeneous optimum bodies with fins, whose bow (nose or leading part) is an optimum cone (OC) and whose stern (aft part) is constructed from segments of an OC and planes tangent to an OC of shorter length. It is shown that for prescribed mass, length, and base area of the body it is always possible to construct a homogeneous optimum body with positive margin of stability of motion. A test of the analytical results was carried out, based on a numerical solution of the Cauchy problem for the system of equations of motion of the body, constructed without simplifying restrictions on the shape of the body or the nature of its motion.

The question of the stability of motion of bodies is closely related to the problem of the construction of bodies possessing minimum drag or maximum penetration depth for rectilinear motion in the medium. An extensive body of literature has been accumulated (see, for example, Refs 1–6) on the modelling of processes of interaction of the medium with the rigid body. Models based on the assumption of the local character of the interaction of the medium with a surface element of the body occupy a special place in the study of processes of penetration of a body into a medium. An annotated list exists, comprising 280 works which employ this assumption.[6]

Assigning local interaction models (LIMs) into a separate class, we note that in the study of high-velocity motion of bodies in dense media, the model enjoying the greatest use is the one in which the pressure on the body surface is described by a binomial formula quadratic in the velocity, with a constant term characterizing the strength of the medium. The tangent stresses are usually written out using the constant friction model or the Coulomb friction model with tangent stresses proportional to the pressure. Their generalization is a mixed friction model proposed for soils[4] in which the tangent stresses are calculated according to the Coulomb friction model if they do not exceed the yield strength of the material of the medium subjected to shear, and are equal to the yield strength in the opposite case.

Exact solutions of problems of the body shape optimization (with respect to drag and with respect to penetration depth) within the framework of the LIM have been obtained[7–11] for prescribed length and base area of the body without simplifying assumptions on its geometry. It has been shown[7–11] that optimum bodies are formed by segments of surfaces whose normal makes a constant angle with the direction of motion, identical for the forward part of the body surface. In general, the optimum angles for bodies of maximum penetration depth and bodies with minimum drag are different. The simplest optimum body is a circular cone. A method of constructing optimum shapes has been proposed,[7–9] using which it is possible to construct an infinite set of optimum bodies with prescribed length and base area, including conical and pyramidal bodies consisting of segments of an optimum cone and planes tangent to it. All these bodies for the same velocity have the same drag and for equal mass have the same penetration depth.

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Optimization of shapes was performed for rectilinear motion of the body. The motion of a body in a medium can be perturbed, and thus, as the results of experimental and theoretical studies have shown,[1–3,5,12–19] the characteristic features of the body shape have a substantial effect on the development of the perturbations and the character of the motion of the body. For high velocities, optimum bodies[7–11] are slender and pointed. The motion of such bodies in a medium is often unstable, and in the presence of perturbations the trajectory of their motion can deviate strongly from the initial direction. Growth of the perturbations can lead to capsizing and disintegration of the body, and thus attainment of the theoretically predicted penetration depth is impossible. An examination of the influence of perturbations on the characteristics of the motion of optimum shapes and a classification of shapes according to stability of motion are important steps in the investigation of the properties of optimum bodies.

Analytical solutions of problems of the dynamics of conical and pyramidal bodies for the above three friction models have been constructed within the framework of the binomial LIM:[15,18] the constant friction model, the Coulomb friction model, and the mixed friction model. Differences in the solutions have been demonstrated[18] and the influence of the model parameters on the motion characteristics of the bodies has been investigated. A criterion of stability of motion of bodies has been obtained,[15,18] and it has been shown that bodies with negative margin of static stability can move stably in a dense medium if their mass is less than some critical value.

It is well known (see, for example, Refs 1 and 3) that the penetration depth of a body increases with increase of its mass. Slender homogeneous optimum conical and pyramidal bodies have negative margin of static stability. Taking into account that for stable motion of such bodies there is a restriction on their mass, we find that the use of these bodies to achieve a penetration depth greater than some limit value can be ineffective. This has been confirmed by numerical calculations[15,16,18] and, in particular, it has been shown that the use of homogeneous steel and tungsten conical bodies for deep penetration into soil media is inadvisable.

It is possible to increase the margin of stability of motion of a body by moving its centre of mass toward the body vertex. In this case, it is possible to increase the margin of stability without modifying the mass of the body or its shape as a result of redistributing the mass of the body over its volume. Thus, for example, it is possible to displace the centre of mass of the body by creating a cavity in its aft part.[5,20,21] Thus, by filling the remainder of its volume with material of greater density, it is possible to conserve the mass of the body and increase its margin of static stability, as has been confirmed theoretically[5,20] and experimentally.[21] However, in this case the homogeneity of the body is breached, which for an asymmetric load upon the body can become the cause of its disintegration.

For homogeneous bodies, the strength characteristics are higher than for inhomogeneous bodies of the same mass. It is possible to increase the margin of stability of motion of a homogeneous body if the centre of pressure of the force acting on it is moved closer to the body base. It is well known that one way of shifting the centre of pressure of the force backward is to equip the body with fins.

Below, a method is developed for constructing homogeneous optimum bodies with fins, where the nose part (bow or leading part) of the bodies contains an optimum cone, and the aft part is constructed from segments of an optimum cone and planes tangent to the optimum cone of shorter length. It is shown that for prescribed mass, length, and base area of the body, it is always possible to construct an optimum body with positive margin of stability of motion. The motion characteristics of constructed bodies were compared with those of a body equivalent to them in mass, length, and base area of the cone with a *skirt*, on whose leading part, as in optimum bodies, there lies an optimum cone, but the aft part belongs to the surface of a different cone with larger opening angle.

1. Model of the interaction between the medium and the body

Let us consider the inertial motion of a body in a medium, assuming that at the initial instant it is completely submerged in the medium and does not deform as it moves. We assume that the influence of the free surface of the medium and also the influence of the gravity force on the motion of the body can be neglected.

We write the force exerted by the medium on the body in the form

$$\mathbf{F} = \iint\limits_S [\sigma_n \mathbf{n} + \sigma_\tau \tau]\, dS \tag{1.1}$$

where σ_n and σ_τ are the normal and tangent stresses on the body surface, and $\mathbf{n}$ and $\boldsymbol{\tau}$ are the unit vectors of the interior normal and the tangent to the surface element, lying in the glide plane of the particles of the medium along the body surface, and the integral runs over the contact surface of the medium with the body S. To write down the stresses, we use binomial formulae, quadratic in the velocity:

$$\sigma_n = A_1(\mathbf{U}\cdot\mathbf{n})^2 + C_1,\ \sigma_\tau = A_2(\mathbf{U}\cdot\mathbf{n})^2 + C_2;\ \mathbf{U} = \mathbf{U}_c + [\boldsymbol{\Omega}\times\mathbf{r}] \tag{1.2}$$

The coefficients A_i and C_i (i = 1, 2) are constant parameters of the model, determined by the characteristics of the medium, $\mathbf{U}$ is the velocity of the surface element, $\mathbf{U}_c$ is the velocity of the centre of mass of the body, $\boldsymbol{\Omega}$ is the angular velocity of rotation of the body about the centre of mass, and $\mathbf{r}$ is the radius vector of the element with its origin at the centre of mass. For plane (longitudinal) motion, when the trajectories of all the particles of the body lie in planes parallel to the plane of motion of the centre of mass, the vector $\boldsymbol{\Omega}$ is normal to this plane and has one non-zero component Ω.

Model (1.2) is a particular case of how the stresses are written within the framework of the LIM. Under certain assumptions (see Refs 1–3, 5, 6) the first of Eqs (1.2) describes the pressure on the surface of the body as it moves in a gas and in dense media of the type soils and metals. The term C_1 characterizes the resistance of the medium to deformation, and A_1 has the order of magnitude of the medium density. For specific media, the values of A_1 and C_1 are found either by solving model problems[1,3,5] or are determined experimentally. Thus, for clayey media, according to the solution obtained for an incompressible elastoplastic medium,[5] it is possible to set

$$A_1 = 3\rho_0/2,\ C_1 = 4\tau_s(1 + \ln(\mu/\tau_s))/3 \tag{1.3}$$

where ρ_0 is the medium density, μ is the shear modulus, τ_s is the yield strength of the material of the medium under shear, and these parameters are constant and do not depend on the velocity.

To write down the tangent stresses σ_τ, it is standard to use the constant friction model or the Coulomb friction model. The constant friction model is often employed to write down σ_τ on the surface of the body under conditions of high-velocity motion in media with low