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Numerical-analytical method for investigating the stability of motion of bodies of revolution in soft soil media☆

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ABSTRACT

A method is presented of investigating the stability of rectilinear motion of a body of revolution in a compressible soil medium with nonlinear physical-mechanical properties of the soil and two-dimensional effects of flow taken into account. The parameters of the axisymmetric process are calculated numerically, whereas the perturbed motion – the radial displacement and rotation relative to the centre of mass – is determined analytically. In the particular case of a conical projectile and linear pressure distribution along the generatrix, an estimate is obtained of the critical position of the centre of mass as a function of the taper angle, the mass and velocity of the body, the coefficient of friction, and the hydrodynamic parameters of the soil medium. Unlike the usually implemented situation of constant pressure postulated by the local interaction models, a displacement of the critical position of the centre of mass by up to 20% of the length of the cone has been found, which leads to a substantial decrease in the margin of stability in a restricted sense. Here, the force parameters and the kinematic parameters of motion of the cone on the boundary of the stability region differ both qualitatively and quantitatively. The stability of motion of bodies in soil media with a nonlinear pressure distribution over the contact surface has not previously been investigated.

Theoretical and experimental studies on the dynamical penetration of bodies into soil media have a long history.[1–4] To date, the description of plane and three-dimensional motions of a rigid body in a resistive medium has been based in the majority of cases on the local interaction theory.[5–10] This mathematical tool, developed in Refs. 11 and 12 and elsewhere, enables one to investigate perturbed motion with asymmetric flow separation taken into account[13–15] as well as the perturbed motion of non-axisymmetric bodies.[16–18]

Numerical and experimental studies demonstrate the applicability of the local interaction model (LIM) to acute (sharply pointed) conical projectiles[19,20] and violation of the conditions of applicability for blunt axisymmetric bodies[21,22] for rectilinear motion of bodies normal to the surface of the soil medium.

In the case of oblique penetration of an acute cone, the force and kinematic parameters obtained within the framework of the LIM, in light of the absence of experimental data, are compared[23] with the data of computer simulations in a three-dimensional statement of the problem. It bears noting that satisfactory qualitative and quantitative agreement of the results (the force vector radial components and the velocity vector, the torque and the angular velocity of rotation about the centre of mass) can be only obtained if the non-constant distribution of contact stresses along the generatrix of the acute cone is taken into account. Despite the fact that the error of the LIM in determining the axial integrated force of resistance to penetration is small, the pressure distribution along the generatrix of an acute projectile and a spherical impactor differs noticeably[20,21] from the result predicted by the LIM. The influence of this effect on the stability of the motion of bodies in soil media has not previously been estimated.

The present paper presents a method of investigating the stability of axisymmetric motion of a body of revolution in a compressible soil medium with nonlinear physical-mechanical properties of the soil[19,24] and two-dimensional effects of the flow[20] taken into account. The parameters of the axisymmetric process are calculated numerically[25] on the basis of the model of an elastoplastic soil medium,[26] and the perturbed motion – radial displacement and rotation about the centre of mass – is determined analytically.[27] In the particular case of a conical projectile and linear stress distribution along the generatrix, a formula is obtained estimating the critical position of the centre of mass as a function of the taper angle, mass and velocity of the body, coefficient of friction, and hydrodynamic parameters of the soil

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medium. Unlike the usually implemented situation of constant pressure postulated by the local interaction models, a displacement of the critical position of the centre of mass by up to 20% of the height of the cone is revealed, which leads to a substantial diminution of the margin of stability in a restricted sense.

1. Equations of motion of the body

The system of equations of three-dimensional motion of the body separates into two independent systems[15,18] describing the motion of the body in two planes. In what follows, we consider plane motion of a rigid body of revolution of length H and base radius R in a moving rectilinear coordinate system Oxy, rigidly bound to the body. The Oy axis coincides with the axis of rotation of the body, and the Ox axis runs perpendicular to it through the body vertex.

The equations of plane motion and rotation of the body about the centre of mass have the form

$$M(\dot{\mathbf{v}} + \boldsymbol{\omega} \times \mathbf{v}) = \mathbf{F}, \quad J\dot{\boldsymbol{\omega}} = \mathbf{K} \tag{1.1}$$

where M is the mass of the body, $\mathbf{F}$ is the principal vector of the forces acting on the body, $\boldsymbol{v} = (v_x, v_y, 0)$ is the velocity vector of the centre of mass, $\boldsymbol{\omega} = (0, 0, \omega)$ is the angular velocity vector of rotation, the vector $\mathbf{K} = (0, 0, K)$ is the angular momentum about the centre of mass, and J is the moment of inertia relative to the axis normal to the plane of motion and passing through the centre of mass; the dot above a symbol denotes differentiation with respect to time.

The force vector $\mathbf{F}$ and the angular momentum vector $\mathbf{K}$ in the equations of motion are defined by integrals of the contact stresses acting on the lateral surface S of the projectile

$$\mathbf{F} = \iint_S [\sigma_n \mathbf{n} + \sigma_\tau \boldsymbol{\tau}]\, dS, \quad \mathbf{K} = \iint_S [(\mathbf{R} - \mathbf{R}_C) \times (\sigma_n \mathbf{n} + \sigma_\tau \boldsymbol{\tau})]\, dS$$

where σ_n and σ_τ are the normal and tangent stresses, $\mathbf{n}$ and $\boldsymbol{\tau}$ are the unit vectors of the interior normal and the tangent to the surface element of the body, $\mathbf{R} = (x, y)$ and $\mathbf{R}_C = (x_C, y_C)$ are the radius vectors of the force application point and the centre of mass.

It is assumed that the flow is unseparated, the projectile is completely submerged in the medium, and the generatrix of the body of revolution is described by the convex function $\{x(y), x(0) = 0, x(H) = R\}$.

The integrals in the relations for the components of the vectors $\mathbf{F}$ and $\mathbf{K}$ are calculated according to the formulae[23]

$$F_x = 2\iint p_x \frac{r}{\cos\eta} d\varphi\, dy, \; F_y = 2\iint p_y \frac{r}{\cos\eta} d\varphi\, dy$$
$$K = 2\iint [(x - x_C)\cos\varphi\, p_y - (y - y_C)\, p_x] \frac{r}{\cos\eta} d\varphi\, dy, \; r(y) = y\,\mathrm{tg}\,\eta, \; \mathrm{tg}\,\eta = \frac{dx}{dy} \tag{1.2}$$

where $r = r(y)$ is the radius of the circle bounding the cross section of the body of revolution intersected by the plane $y = \text{const}$, φ is the angle in this cross section, measured from the positive direction of the Ox axis, $0 \le \varphi \le \pi$, $p_x = p_x(\varphi, y)$ and $p_y = p_y(\varphi, y)$ are functions defining the radial and axial components of the stress vector in an element of the body formed by the angle φ and the coordinate y:

$$p_x = (\cos\eta\,\sigma_n + \sin\eta\,\sigma_\tau)\cos\varphi\,, \; p_y = -\sin\eta\,\sigma_n + \cos\eta\,\sigma_\tau \tag{1.3}$$

and σ_n and σ_τ are the normal and tangent stresses. Here and everywhere below, if not stated otherwise, integration over y runs from 0 to H, and integration over φ runs from 0 to π.

Assuming that the stress for $v_x = \omega = 0$ is described by the dependence $\sigma_n = \sigma(v_n)$, where $v_n = (V, \mathbf{n})$ is the normal component of the velocity vector of the body $\mathbf{V} = \mathbf{v} + \omega \times (\mathbf{R} - \mathbf{R}_C)$, in what follows we consider the approximation

$$\sigma_n \approx \sigma_0 + \sigma' \frac{\partial v_n}{\partial v_x} v_x + \sigma' \frac{\partial v_n}{\partial \omega}\omega, \quad \sigma' \equiv \frac{\partial \sigma}{\partial v_n} \tag{1.4}$$

for arbitrary small perturbations v_x and ω.

In the element of the body designated by the radius vector (x, y) and the corresponding angle φ, the normal component of the velocity vector is equal to

$$v_n = (v_x - (y - y_C)\omega)\cos\varphi\cos\eta - (v_y + (x - x_C)\omega\cos\varphi)\sin\eta \tag{1.5}$$

We introduce the notation

$$f_1(\eta) = 1 - k_f\,\mathrm{tg}\,\eta, \quad f_2(y,\eta) = y_C - \frac{y}{\cos^2\eta}, \quad f_3(y,\eta) = (1 - k_f\,\mathrm{tg}\,\eta)\, y_C - \frac{y}{\cos^2\eta} \tag{1.6}$$

Integrating in relations (1.2) over the angle φ and taking formulae (1.3)–(1.6) into account, we obtain

$$F_x = \pi\int \sigma' x(y) f_1(\eta)\cos\eta (v_x + f_2(y,\eta)\omega)\, dy, \quad F_y = 2\pi\int (1 + k_f\,\mathrm{ctg}\,\eta)\,\mathrm{tg}^2\eta\,\sigma_0 x(y)\, dy$$
$$K = \pi\int \sigma' (v_x + f_2(y,\eta)\omega)\, x(y) f_3(y,\eta)\cos\eta\, dy \tag{1.7}$$

Here it has been assumed that the tangent stress is described by the Coulomb friction law $\sigma_\tau = k_f \sigma_n$ with constant coefficient of friction k_f.