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Characteristic features of the process of wear of a punch and a thin elastic strip with non-uniform friction and wear parameters☆

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ABSTRACT

An analytical solution of the problem on the wear of a punch sliding on a thin elastic strip under the condition that they are in complete contact is given for variable friction and wear coefficients. The particular case of a wavy punch (periodic problem) is considered. Some distinguishing features of the process of wear in the transient regime are revealed. It is established, for example, that the contact pressure distribution curve can have a narrow local maximum (peak), significantly exceeding the average level of the contact pressure in magnitude.

The contact problem in the presence of wear for bodies with variable resistance to wear was first considered in application to an elastic half-plane.[1] A characteristic aspect of the process of the wear of such bodies was established, namely, the existence of a steady-state regime in which the worn surface profile and the contact pressure take on asymptotic distributions. The evolution of the profile of a surface with variable resistance to wear was also investigated in the case of a massive counter-body with a viscoelastic restraint sliding along the surface.[2]

Below, we consider the process of wear of a wavy punch using a simplified model of deformation of a Winkler-type strip with a correction for friction. This allows us to obtain a partial differential equation for the wear, admitting an analytical solution. Special attention is given to the transient regime (running-in) preceding the steady-state regime of wear of the friction pair under consideration.[3] The effect of the combined influence of variable friction and wear coefficients on the process of wear is investigated.

1. Problem statement and the main equations

Let us consider sliding contact of an absolutely rigid body (punch) and an elastic strip of width h, bonded to an absolutely rigid foundation (Fig. 1). We introduce the coordinate system Oxy, the x axis of which we cause to coincide with the foundation boundary and we allow the foundation to move together with the strip with a constant velocity opposite to the x axis, wherewith the coordinate system Oxy remains in place. The punch does not move along the x axis, but has the ability to move along the y axis. It is assumed that the punch is in contact with the strip over its entire length (complete contact) that ensures satisfaction of the inequality

$$0 < p(x,t), \quad x \in (-\infty, \infty) \tag{1.1}$$

It will be verified below for the found contact pressure distributions $p(x, t)$.

The relative sliding of the punch and the strip is accompanied by friction between them and also by wear of the punch. Suppose the tangential boundary stress generated by the friction $\tau = \tau_{xy}\big|_{y=h}$ is related to the contact pressure $p = -\sigma_y\big|_{y=h}$ by the Amonton–Coulomb friction law[4]

$$\tau(x,t) = \mu(x)p(x,t) + \tau_0, \quad \mu(x) > 0 \tag{1.2}$$

where $\mu(x)$ is the coefficient of friction, τ_θ is the adhesion component of friction. The rate of the linear wear W of the punch moving with constant sliding velocity is defined by the contact pressure according to the wear law[4].

$$\dot{W}(x,t) = \alpha(x)p(x,t), \quad \alpha(x) > 0 \tag{1.3}$$

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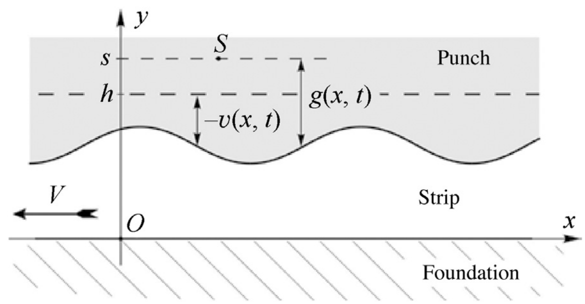

Fig. 1.

where $\alpha(x)$ is the coefficient of wear. Here and below, a dot above the symbol of a function denotes the partial derivative with respect to t whereas a prime will denote the partial derivative with respect to x. We assign the time $t = 0$ to the onset of wear and set $W(x, 0) = 0$.

We choose some point S of the punch and denote its ordinate by s (Fig. 1). We will use the distance $g(x, t)$ between the punch boundary and the level $y = s(t)$ to describe the punch shape varying during the process of wear. By virtue of the above-imposed restrictions on the punch motion, the coordinate of the point S remains fixed. In what follows, we assume the punch profile to be mildly inclined: $|g'(x, t)| \ll 1$. This, in particular, allows us to identify the wear W of the punch with the motion of its boundary along the y axis as the result of wear.

Under the action of the punch, the strip deforms elastically. The corresponding displacement v of the strip upper boundary in the y direction is related to the punch shape by the equality $-v(x, t) = g(x, t) - (s(t) - h)$, where

$$g(x,t) = g_0(x) - W(x,t) \tag{1.4}$$

$s(t) = s_0 - \delta(t)$, $\delta(t)$ is the punch penetration into the strip during the process of wear, $\delta(0) = 0$. Here and below, a zero subscript denotes the value of a function at the initial time $t = 0$. Without loss of generality, in what follows we set $s_0 = h$. All this allows us to obtain the following condition for contact of the punch with the strip:

$$-v(x,t) + W(x,t) = g_0(x) + \delta(t) \tag{1.5}$$

Assuming the strip to be asymptotically thin[5] and taking into consideration friction law (1.2), we will use a simplified model of the strip deformation[6]

$$Bhp(x,t) = -v(x,t) - \lambda(x)v'(x,t) \tag{1.6}$$

where $B = (1-2\nu)[2G(1-\nu)]^{-1}$, $\lambda(x) = (1-4\nu)h[2(1-2\nu)]^{-1}\mu(x)$, G and ν are the shear modulus and Poisson's ratio of the strip material, and $\nu \in [0, 1/2]$. According to relation (1.6), the adhesion component τ_0 of friction has no effect on the contact pressure. This property is characteristic of problems, the statement of which assumes complete contact of the interacting bodies.[6,7]

The degree of loading of the contact of the punch with the strip is characterized by the specific load, coinciding with the mean value of the contact pressure

$$\bar{p}(t) = \lim_{z\to\infty} \frac{1}{2z} \int_{-z}^{z} p(x,t)\, dx \tag{1.7}$$

Using contact condition (1.5), we eliminate the displacement v from relation (1.6) and as a result we arrive at an expression for the contact pressure

$$p(x,t) = \frac{1}{Bh}\left[\varphi(x,t) - W(x,t) - \lambda(x)W'(x,t)\right] \tag{1.8}$$

$$\varphi(x,t) = g_0(x) + \lambda(x)g_0'(x) + \delta(t) = Bhp_0(x) + \delta(t) \tag{1.9}$$

In the latter equality we made use of the expression

$$p_0(x) = \frac{1}{Bh}\left[g_0(x) + \lambda(x)g_0'(x)\right] \tag{1.10}$$

which follows from formula (1.8) for $t = 0$. In what follows we assume that the functions $g_0(x)$ and $\lambda(x)$ ensure satisfaction of the condition of complete contact (1.1) at the initial time.

On the other hand, the contact pressure can be expressed in terms of the wear using wear law (1.3). Thus, from equality (1.8) we obtain the following equation for the wear:

$$\lambda(x)W'(x,t) + \frac{1}{r(x)}\dot{W}(x,t) + W(x,t) = \varphi(x,t); \qquad r(x) = \frac{\alpha(x)}{Bh} > 0 \tag{1.11}$$

The actual shape of the punch $g(x, t)$ is determined by its wear $W(x, t)$ in accordance with formula (1.4).

We pose the problem: from the assigned initial shape of the punch $g_0(x)$ and its penetration $\delta(t)$, to find the functions $W(x, t)$ and $p(x, t)$ describing the process of wear of the punch interacting with the strip.

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