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Optimization of the flight trajectory of a non-manoeuvrable aircraft to minimize fuel consumption by the dynamic programming method☆

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ABSTRACT

Using Bellman's dynamic programming method, a fuel-consumption optimum flight trajectory of a typical mid-range aircraft is constructed for a prescribed range. The optimum trajectory was calculated for the full model of motion of the aircraft along a trajectory in a vertical plane. Optimization was realized simultaneously for the whole trajectory without decomposing the process into individual steps. The method made it possible to take into account all restrictions imposed both by technical peculiarities of the aircraft and also by other requirements – safety and comfort of the passengers.

In connection with the fact that transport aircraft, unlike fighter jets, are characterized by low manoeuvrability, individual segments of the flight trajectory – climb, change of level, descent, motion along a glide path – have been investigated relatively deeply. The problem of optimization of the trajectory as a whole to minimize fuel consumption is of practical interest.

Aircraft trajectory control algorithms should ensure satisfaction of conditions of safety and economy, and at the same time require minimum computation time. Below, we consider the solution of the problem of minimum fuel consumption during flight from one point to another. Similar problems have been solved by decomposition of the trajectory into three parts: cruise climb, cruising segment, and descent with approach to landing.[1–8] The problem of minimizing fuel consumption (or the generalized criterion consisting of a linear combination of fuel consumption and flight time) was solved on each segment separately. Of course, the mutual influence of adjoining segments was taken into account. To construct the entire flight trajectory, the solutions obtained on individual segments are joined together using some intermediate segments.

In the optimization of the segments, an energy approach was applied to describe the motion of the aircraft, apparently first proposed in Ref. 9 for the optimization of trajectories of flying machines and then widely used to solve different problems of optimization of trajectories, in particular, to solve the problem of four-dimensional navigation.[1–5] As a rule, when using this optimization approach, Pontryagin's maximum principle is applied.[10]

The need arises to check the results obtained by the indicated method by optimizing the trajectory as a whole without subdividing the problem into individual tasks. An exact solution by methods proposed earlier[1] in the presence of complicated and numerous phase constraints is exceedingly difficult. Bellman's dynamic programming method makes it possible to take numerous and complicated constraints into account, both those imposed by technical peculiarities of a definite type of aircraft and those arising from other requirements, for example, safety and passenger comfort.

Many recent studies on optimization of trajectories of non-manoeuvrable aircraft[2–18] have been based on significantly simplified models of motion, not taking phase and other constraints into account. In this connection, a study including a significant number of elements inherent in real-world conditions would be of interest. The present paper is dedicated to that goal.

1. Statement of the problem

The main role in ensuring economization is played by control of the aircraft in the vertical plane. We will assume that the flight trajectory is always found strictly in the vertical plane. The coordinates of the centre of mass of the aircraft x, y are assigned in a fixed right-handed

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Cartesian coordinate system OXY (the Y axis is directed along the vertical and the X axis is horizontal). Then, the equations of motion of the aircraft have the form[19]

$$\frac{dV}{dt} = g(n_x - \sin\theta), \quad \frac{d\theta}{dt} = \frac{g}{V}(n_y - \cos\theta)$$
$$\frac{dy}{dt} = V\sin\theta, \quad \frac{dx}{dt} = V\cos\theta, \quad \frac{dm}{dt} = -Q_t(M, y, P) \tag{1.1}$$

Here, g is the acceleration due to gravity, V is the magnitude of the aircraft velocity, m is its mass, θ is the pitch angle, n_x is the tangential load, n_y is the load factor, Q_t is the fuel consumption per second, which depends on the Mach number M, the flight altitude y, and the thrust P.

We will neglect the normal component of the tractive force. Thus, for the load factor n_y and the tangential load n_x the following expressions are valid:

$$n_y = \frac{C_y qS}{mg}, \quad n_x = \frac{P}{mg} - \frac{qSC_x}{mg}; \quad q = q(y, V) = \frac{\rho(y)V^2}{2} \tag{1.2}$$

where q is the dynamic pressure, $\rho(y)$ is the mass density of the atmosphere as a function of altitude, S is the wing area, taken in what follows to be equal to 168.3 m^2, C_x is the drag coefficient, which is a function of the lift coefficient C_y and the Mach number M.

Eliminating time from the equations of motion by using the fourth of Eqs (1.1), we obtain

$$\frac{dV}{dx} = \frac{g}{V}\left(\frac{n_x}{\cos\theta} - \operatorname{tg}\theta\right)$$
$$\frac{d\theta}{dx} = \frac{g}{V^2}\left(\frac{n_y}{\cos\theta} - 1\right), \quad \frac{dy}{dx} = \operatorname{tg}\theta, \quad \frac{dm}{dx} = -\frac{Q_t(M, y, P)}{V\cos\theta} \tag{1.3}$$

The controlling parameters are the lift coefficient C_y and the magnitude of the tractive force P. Substituting the expression for n_y in relations (1.2) into the second of Eqs (1.3), we find

$$C_y = \frac{2m\cos\theta}{\rho(y)S}\left(\frac{d\theta}{dx} + \frac{g}{V^2}\right) \tag{1.4}$$

Substituting the expression for n_x (1.2) into the first of Eqs (1.3), we have

$$P = \frac{1}{2}\rho(y)V^2 SC_x + mg\sin\theta + mV\cos\theta\frac{dV}{dx} \tag{1.5}$$

It follows from system (1.3) that to describe the motion of the object it suffices to know the three functions $V(x)$, $\theta(x)$, and $y(x)$, on the basis of which it is possible to find the control functions (parameters) $C_x(x)$ and $P(x)$ from formulae (1.4) and (1.5), and also the aircraft mass $m(x)$ by bringing to bear the last equation of system (1.3).

The flight time T is not fixed.

The initial and final positions of the aircraft are assigned as follows:

$$t_0 = 0: \; x(0) = 0, \; y(0) = y_0 = 100 \text{ m}, \; m(0) = m_0 = 7\cdot10^4 \text{ kg}, \; V(0) = V_0 = 100 \text{ m/s}, \; \theta(0) = 0$$
$$t_T = T: \; x(T) = L = 5\cdot10^5 \text{m}, \;\; y(T) = 600 \text{ m}, \;\; V(T) = V_k = 100 \text{ m/s}, \;\; \theta(T) = 0$$

It is required to choose the control parameters such that the aircraft travels from the prescribed initial position to the prescribed final position with minimum fuel consumption. The optimization criterion has the form

$$\int_0^T Q_t dt \to \min \quad \text{or} \quad J = \max_{C_y, P} m(T)$$

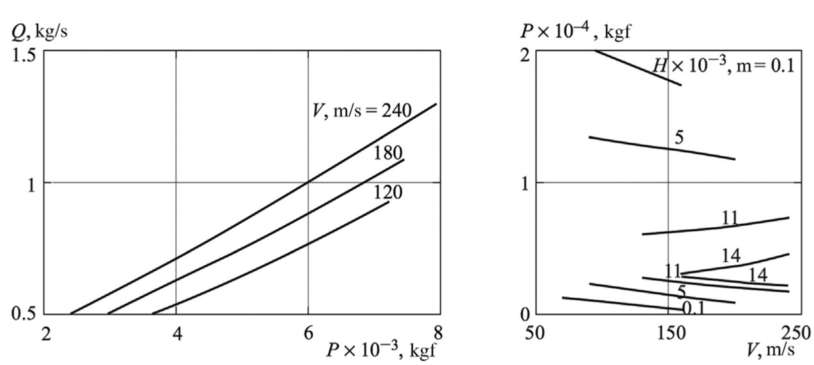

Fig. 1.