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Asymmetric separated flow about a Nikolsky wing with a protrusion☆

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ABSTRACT

The flow of an inviscid incompressible fluid about a wing of low aspect ratio with a parabolic planform (a Nikolsky wing) is investigated; a protrusion, whose height grows according to a parabolic law, is mounted on the leeward side of the wing in the symmetry plane. It is shown that for symmetric boundary conditions, along with a symmetric solution, an asymmetric solution also exists. The dependence of the asymmetric solution on the wing geometry is examined. It is shown that critical values of the wing curvature and the height of the protrusion exist, for which the asymmetric solution continuously transitions to the symmetric one. In addition, it is shown that a limit asymmetric solution exists which corresponds to an infinitely large protrusion. The stability of the solutions found is discussed.

In the motion of a body in an inviscid incompressible fluid, separation of the vorticity from the body surface in the form of vortex sheets (surfaces of tangential discontinuity of the velocity) or vortex filaments is possible.[1–7] For bodies of low aspect ratio, the three-dimensional steady problem in the leading approximation reduces to a planar, time-dependent problem.[2] A planar, temporally self-similar problem of a plate extending outward from a point according to a power law and moving according to the same law was considered in Refs 3 and 4. It was found that for a limit value of the self-similarity index equal to 0.5, vortex sheets, separating from the plate edges, transition to discrete vortices, which are generated at the initial time, and then they control the flow in such a way that flow about the plate edges becomes unseparated. This problem corresponds to flow about a wing of low aspect ratio having a parabolic shape in the planform and parabolic curvature (a Nikolsky wing). According to the solution given in Ref. 3, two vortex filaments of opposite intensity separate from the vertex of the Nikolsky wing. This result was later experimentally confirmed.[8]

Issues of non-uniqueness and asymmetry of the solutions have been considered.[5–7,9,10] Flow about a conical fuselage with triangular wings, located under the angle of attack has been considered.[5,6,10] For a triangular plate located under the angle of attack and having on the leeward side in the symmetry plane a triangular separating plate, it was shown[7,9] that for the angle of attack exceeding some critical value the symmetric solution is unstable, and a stable asymmetric solution is realized. In this case, asymmetric vortex sheets separate from the edges, which leads to the appearance of a side force acting on the body. It should be noted that flow about a triangular wing without a protrusion under symmetric boundary conditions is always symmetric. In the opposite case, a side force that should be applied to a finite area will act on the wing. Results of calculations[7] are in qualitative agreement with experiment.[9]

The vortex sheet was modelled[6] by a discrete vortex joined to the edge of the wing by a cut. To describe the outer part of the vortex sheet, the method of discrete vortices was used,[7] and also a finite-difference representation of the integro-differential equation of the evolution of the vortex sheet, which was solved by the method of iterations.[5,10] The vortex–cut model was used to describe the core of the vortex sheet.[5,7,10] In addition, a 3D-RANS CFD calculation was performed,[10] taking viscosity into account. A substantial difference in the solutions taking and not taking viscosity into account at high angles of attack was noted. A review of the enumerated results is contained in the Ref. 11 (monograph).

The present work investigates flow about a Nikolsky wing with a plate mounted on the leeward side in the symmetry plane, where the plate height grows according to a parabolic law. The presence of the protrusion enables the existence of asymmetric solutions. By virtue of the limit value of the self-similarity index, the asymmetric solution consists of three vortex filaments separating from the vertex of the wing.

Unlike to studies of asymmetric flows by approximate methods,[6,7] an exact solution is obtained below. Moreover, separation on the protrusion is examined, which was not done in Ref. 7, in which the protrusion to be blunt was assumed. Another difference between the

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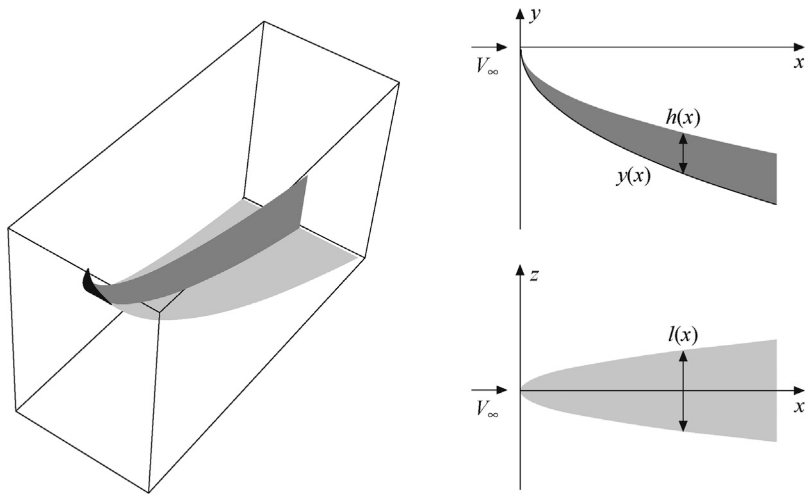

Fig. 1. Geometry of a Nikolsky wing with a protrusion.

problem considered here and previous problems[6,7] is the absence of a fixed angle of attack (the angle of attack is equal to 90° at the tip of the wing and 0° at infinity).

1. Statement of the problem

The geometry of a Nikolsky wing with a protrusion (Fig. 1) is described by the relations

$$l(x) = 2\varepsilon\alpha x^n, \quad y(x) = -\varepsilon\beta x^n, \quad h(x) = \varepsilon\gamma x^n \tag{1.1}$$

Here $l(x)$ is the dependence of the width of the wing on the longitudinal coordinate, $y(x)$ is the intersection line of the wing and the protrusion, $h(x)$ is the protrusion height, ε is a scaling parameter, α, β, and γ are dimensionless constants, and n is the self-similarity index, $0 \le x < \infty$, $0.5 \le n \le 1$. Below, we consider the limit value of the self-similarity index $n = 0.5$. The case $n = 1$, corresponding to a triangular wing with a triangular protrusion, was considered earlier.[7]

We investigate separated flow of inviscid, incompressible fluid about a Nikolsky wing with a protrusion, the steady, uniform free stream is directed along the x axis and has velocity V_∞. The dependence of the solution on two dimensionless parameters characterizing the wing geometry – the relative height of the protrusion b (ratio of the height to the half-width) and the curvature of the wing a_0 (ratio of the depth of the sag to the half-width)

$$b = \gamma/\alpha, \quad a_0 = \beta/\alpha \tag{1.2}$$

is examined.

The solution should satisfy boundary conditions of impermeability at the surface of the rigid body, the Chaplygin–Zhukovsky conditions at the edges, and the condition of unperturbed flow in the limit of infinite distance from the wing.

2. Solution method

The unsteady analogy is valid for $x \gg \varepsilon x^n$ or $x \gg \varepsilon^{1/(1-n)}$, when it is possible to assume that we have the low aspect ratio wing. Within the framework of the unsteady analogy, the problem reduces to the study of motion of an expanding plate with a protrusion. Here, the law describing the expansion of the plate and its motion along the y axis is found by substituting the relation $x = V_\infty t$ into formulae (1.1) taking note of the notation defined by relations (1.2):

$$l(t) = 2at^n, \quad h(t) = abt^n, \quad y(t) = -aa_0t^n; \quad a = \varepsilon\alpha V_\infty{}^n$$

We choose a coordinate system with its origin at the base of the protrusion. We direct the x axis along the plate and the y_1 axis along the protrusion. The velocity of the flow along the y_1 axis in this coordinate system is equal to

$$-dy/dt = naa_0t^{n-1}$$

The problem under consideration is self-similar since there is no characteristic dimension in it and no characteristic time interval. The circulation of vortex formations separating from the sharp edges of the plate and the protrusion varies proportionately to t^{2n-1} (Ref. 1). As the self-similarity index n tends to its limit value 0.5, the circulation ceases to depend on time, there is no continuous separation of the vortex sheet from the edges of the plate is absent, and three discrete vortices are formed in the flow, which separate from the edges at the initial instant of time $t = 0$. Everywhere below we assume that $n = 0.5$.

We introduce a complex coordinate in the physical plane $z_1 = x_1 + iy_1$ and in the self-similarity plane $z = x + iy$, where $z_1 = at^{1/2}z$. We relate the velocity and circulation of the vortical formation in the physical and self-similar variables as follows:

$$\mathbf{u}_1 = at^{-1/2}\mathbf{u}, \quad \Gamma = a^2G$$