



Application of the theory of reversible discontinuities to the investigation of equations describing waves in tubes with elastic walls[☆]

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ABSTRACT

Equations which describe the propagation of waves in tubes with elastic walls are investigated, methods for calculating them are developed, and solutions containing reversible discontinuity structures are analysed in the case of a fluid-filled tube. The model of a tube with elastic walls constructed on the basis of the complete model of a membrane and the non-linear theory of elasticity is generalized. The viscosity and compressibility of the material, the possibility of filling the tube with a gas, and the flexural rigidity of the tube walls are taken into account. The problem of the decay of an arbitrary discontinuity is solved numerically in the case of a fluid-filled tube. The results obtained correspond to the previously developed theory of reversible discontinuities. Simplified hyperbolic equations of long waves, as well as equations for small-amplitude waves which do not take into account longitudinal elastic waves and are similar to the Boussinesq equations, are derived for cases when a tube is filled with a liquid and a gas. The possibility of overturning of the waves is analysed. A procedure for correcting the numerical schemes by adding terms with high-order derivatives to the equations is developed, and the order of approximation of the numerical scheme remains unchanged, enabling the performance of calculations with low schematic dissipation.

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1. Introduction

Equations which describe flow in tubes with elastic walls have been considered in many studies owing to the relevance of such investigations for technical applications and for the study of biological objects, as described, for example, in the review in Ref. 1, where, in particular, model equations with dispersion, namely, the Korteweg–de Vries equation (the KdV equation) and the non-linear Schrödinger equation, were used. Here we will examine equations obtained on the basis of a complete model of a membrane and a non-linear elastic model with finite strains using the initial Lagrangian longitudinal coordinate.² These equations have been investigated analytically in the case of controlled pressure³ and in the case when a tube is filled with a fluid.^{4–6} The main purpose of these studies was to investigate solitary waves.

The theory of reversible discontinuities^{7–11} was developed for models with complex dispersion. Its main elements are prognosis of the possible type of discontinuity from the dispersion relation and classification of the discontinuity structures. According to this theory, the solutions should contain homogeneous zones, centred simple waves and time-expanding wave zones (expanding discontinuity structures), which can be described by averaged equations, as well as local discontinuity structures in the form of classical and generalized kinks and solitary waves. In addition, weakly dissipative dispersion models were investigated. In this case, the wave zones do not expand with time, but they are also described by averaged equations. The solitary waves and discontinuity structures in the case of a tube with controlled pressure were investigated in detail by analytical methods and by methods of numerical analysis;¹² solutions of the Riemann problem (the problem of the decay of an arbitrary discontinuity) were considered, and it was concluded that the solutions correspond to the theory of reversible discontinuities.

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The purpose of the present work is to investigate equations constructed on the basis of the complete membrane model and the non-linear theory of elasticity by applying the theory of reversible and weakly dissipative discontinuities to them. In view of the fact that the equations were previously investigated in the case of a tube with controlled pressure, here the equations are investigated mainly in the case when the tube is filled with a liquid or a gas. A generalization of the known equations is proposed: the viscosity and compressibility of the tube material and the compressibility of the filler are taken into account, and the flexural rigidity of the tube walls is taken into account. This is done to determine the boundaries of the applicability of the results obtained and of the calculation methods. Then the solutions of the problem of the decay of an arbitrary discontinuity in the case of a fluid-filled tube are analysed. The principal structures of reversible discontinuities that were previously revealed for other models are observed in the solutions. Next, simplified equations of the hyperbolic type and equations of low-amplitude long waves, which are similar to the ordinary and generalized Boussinesq equations, are derived for the cases of filling with a liquid and a gas. The structures of discontinuities were already previously investigated for the Boussinesq equations, enabling us to compare the results. Then the possibility of reversal of the waves is discussed, and it is concluded that in the present model, unlike the other models, dispersion is generated by terms with low-order derivatives, and therefore, despite the presence of discontinuity structures, reversal is possible for some initial data. The numerical methods used to solve the equations that describe flows in tubes are presented in the conclusion. The results of the theory of reversible and weakly dissipative discontinuities were used to select the numerical scheme by analysing the solutions obtained for the Riemann problem. A procedure for correcting the schemes by adding terms with high-order derivatives, which enables us to achieve convergence without lowering the order of the approximation, was used in the problem of a fluid-filled tube.

2. Basic equations and generalizations of the model

The equations of motion of waves in a tube with incompressible axisymmetric elastic walls and fixed internal and external pressures have the form³

$$\left(R\sigma_1 \frac{z'}{\lambda_1^2} \right)' - P^* r r' = \rho R \ddot{z}, \quad \left(R\sigma_1 \frac{r'}{\lambda_1^2} \right)' - \frac{\sigma_2}{\lambda_2} + P^* r z' = \rho R \ddot{r}$$

$$\lambda_1 = \sqrt{r'^2 + z'^2}, \quad \lambda_2 = \frac{r}{R}, \quad \lambda_3 = \frac{h}{H}, \quad P^* = \frac{P}{H}, \quad \sigma_i = \lambda_i \frac{\partial W}{\partial \lambda_i} - p, \quad i = 1, 2, 3 \quad (2.1)$$

A prime denotes differentiation with respect to the variable Z , which is the initial Lagrangian spatial coordinate along the tube, a dot denotes differentiation with respect to the time t , H and h are the thickness of the tube wall in the stress-free and stress states, $W = W(\lambda_1, \lambda_2, \lambda_3)$ is the elastic potential, p is the pressure associated with the incompressibility of the tube material,¹³ which is similar to the pressure in an incompressible fluid (in the theory of the elasticity of incompressible materials, this quantity is called the Lagrange multiplier^{14,15}), the parameter P is the difference between the internal and external pressure, and the parameter ρ is the density of the tube material per unit area. The elastic properties of the tube wall are described using the membrane model, the unknowns z and r assign the tube surface in a cylindrical coordinate system, and the z axis of this system coincides with the tube axis. Here the λ_i are the principal stretches, and the σ_i are the principal stresses, that is, the stress tensor components. The subscripts 1, 2, 3 correspond to the latitudinal (circumferential), meridional (tangential in a plane passing through the rotation axis) and orthogonal directions of the deformable surface. It is assumed that when there is no load we have

$$z = Z, \quad r = R, \quad h = H$$

A model with controlled pressure is suitable in the case in which the tube is filled with a gas of negligibly small density. The pressure is maintained constant by the flow of the medium into the tube from a reservoir of large volume or the operation of a compressor.

In the case of the use of a viscoelastic model of the Kelvin–Voigt type,¹⁶ it is logical to add viscous stresses, such as, for example,

$$\sigma_i \rightarrow \sigma_i + \sigma_{vi}, \quad \sigma_{vi} = \nu_{vi} \dot{\lambda}_i, \quad i = 1, 2, 3$$

The viscosity coefficients can depend on the strain.

The tube material is assumed to be incompressible; therefore,¹⁷

$$\lambda_1 \lambda_2 \lambda_3 = 1, \quad \sigma_i = \lambda_i \hat{W}_i, \quad \hat{W}(\lambda_1, \lambda_2) = W(\lambda_1, \lambda_2, 1/(\lambda_1 \lambda_2)), \quad i = 1, 2$$

Here and below, the notation $\hat{W}_i = \partial \hat{W} / \partial \lambda_i$ is used. The formula presented above for the stresses σ_1 and σ_2 is derived from the assumption that the transverse stresses σ_3 are equal to zero (the membrane model). More precisely, it is assumed that the quantities p and $\lambda_3 W_3$ are large compared with the pressure difference P . In the case when the viscoelastic model described above is used, in a similar manner we obtain

$$\sigma_{vi} = \nu_{vi} \dot{\lambda}_i - \nu_{v3} [1/(\lambda_1 \lambda_2)], \quad i = 1, 2$$

In the case when the compressibility of the material is taken into account and p is determined by the elastic strains (see Section 3), formulae that do not contain λ_3 can also be derived for σ_1 and σ_2 by taking advantage of the assumption that $\sigma_3 = 0$ and finding λ_3 in terms of λ_1 and λ_2 . For example, for the Hencky material described in Section 3, for which

$$\sigma_i = \lambda [1 - \lambda / (\lambda + 2\mu)] \ln(\lambda_1 \lambda_2) + 2\mu \ln \lambda_i, \quad i = 1, 2$$

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