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Group analysis of a one-dimensional model of gas filtration[☆]

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ABSTRACT

A potential has been introduced with the help of a conservation law for the simplest one-dimensional model of gas filtration in a porous medium. The admissible Lie algebra of this model with this potential is extended by a new transport operator. An optimal system of non-similar subalgebras has been constructed. For one-dimensional subalgebras, all invariant sub-models have been considered and the solutions have been investigated qualitatively. Group analysis can be extended by a consideration of differentially invariant sub-models for subalgebras of greater dimensionality.

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A model of gas filtration in a porous medium has been obtained from the mass conservation law, the equation of state of the gas, and the filtration law¹

$$(m\rho)_t + \nabla \cdot (\rho \mathbf{u}) = 0, \quad \rho = \rho(p, T), \quad \mathbf{u} = -k\mu^{-1} \nabla p$$

where m is the porosity, ρ and μ are the density and viscosity, p is the pressure, T is temperature, k is the permeability, and u is the filtration rate. In the simplest case, when the gas is thermodynamically ideal with constant viscosity, and for isothermal motion, the model reduces to the equation of piezoconductivity²

$$p_t = k(2m\mu)^{-1} \Delta p^2$$

For this equation, self-similar solutions have been considered which were obtained by the dimensionality method,³ and also solutions which are the limits of these self-similar solutions.¹ The modern theory of finding such solutions is group analysis of differential equations.^{4–6} Elements of group analysis for parabolic evolution equations have been derived⁷: equivalence transformations and new admissible operators of some equivalent equations, and examples of exact solutions have been considered. Below, we perform a group analysis of the one-dimensional equation of piezoconductivity (all invariant sub-models are considered) with the aim of systematizing previously obtained results,^{1,7} and also to present possibly new invariant solutions and describe their properties.

1. Subalgebras of the one-dimensional model

In the one-dimensional case, the extension $t \rightarrow t\mu mk^{-1}$ reduces the equation of piezoconductivity to its dimensionless divergent form

$$p_t = (pp_x)_x \quad (1.1)$$

It is possible to introduce the potential

$$p = \varphi_x, \quad pp_x = \varphi_t \quad (1.2)$$

By direct calculation according to a previously described algorithm^{4,5} it has been proven that system (1.2) admits a five-dimensional Lie algebra L_5 with basis consisting of the operators

$$X_1 = \partial_x, \quad X_2 = \partial_t, \quad X_3 = \partial_\varphi, \quad X_4 = 2t\partial_t + x\partial_x + \varphi\partial_\varphi, \quad X_5 = -t\partial_t + p\partial_p + \varphi\partial_\varphi$$

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Introducing the potential extends the Lie algebra of Eq. (1.1) by one basis operator: the transformations of variables admitted by system (1.2) consist in transports and extensions.

The nonzero commutators of the basis operators are

$$[X_1, X_4] = X_1, \quad [X_2, X_4] = 2X_2, \quad [X_2, X_5] = -X_2, \quad [X_3, X_4] = X_3, \quad [X_3, X_5] = X_3$$

The internal automorphisms satisfy the operator equation

$$\partial_a X' = [X_i, X'], \quad X'|_{a=0} = X = x^j X_j, \quad i, j = 1, 2, 3, 4, 5$$

Hence we obtain the following automorphisms (variables for which the transformations are not written down are invariant):

$$\begin{aligned} 1) x'_1 &= x_1 + a_1 x_4, \quad 2) x'_2 = x_2 + a_2 (2x_4 - x_5), \quad 3) x'_3 = x_3 + a_3 (x_4 + x_5) \\ 4) x'_1 &= ax_1, \quad x'_2 = a^2 x_2, \quad x'_3 = ax_3, \quad 5) x'_2 = bx_2, \quad x'_3 = b^{-1} x_3 \end{aligned}$$

The discrete automorphisms

$$6) x'_2 = -x_2, \quad 7) x'_1 = -x_1, \quad 8) x'_3 = -x_3$$

are noted.

The Lie algebra L_5 is decomposed into a semidirect sum of the Abelian subalgebra $\{X_4, X_5\}$ and the Abelian ideal $\{X_1, X_2, X_3\}$. To within internal automorphisms, we list all the subalgebras. Since the elements of the Abelian subalgebra do not transform, its subalgebras are

$$\{0\}, \{X_4 + \alpha X_5\}, \{X_5\}, \{X_4, X_5\}$$

For each subalgebra of the Abelian subalgebra, an arbitrary element of the Abelian ideal is added to each element of the basis, and the element of the Abelian ideal is assigned so as to form a subalgebra of zero dimensionality. The representation so obtained is simplified by internal automorphisms, and the conditions of the subalgebra are verified. As a result, an optimal system is obtained, where $\delta = 0$ or 1 , $\varepsilon = 0$ or 1 , and α is an invariant of the automorphisms.

The one-dimensional subalgebras are

$$\begin{aligned} \{X_1 + \varepsilon X_2 + \delta X_3\}, \{X_2 + \varepsilon X_3\}, \{X_3\}, \{X_4 + \alpha X_5, \alpha \neq 2, -1\} \\ \{X_4 + 2X_5 + \varepsilon X_2\}, \{X_4 - X_5 + \varepsilon X_3\}, \{X_5 + \varepsilon X_1\} \end{aligned} \quad (1.3)$$

The two-dimensional subalgebras are

$$\begin{aligned} \{X_1 + \alpha X_3, X_2 + \varepsilon X_3\}, \{X_1 + \varepsilon X_2, X_3\}, \{X_2, X_3\}, \{X_4 + \alpha X_5, X_1\} \\ \{X_4 + X_5, X_1 + X_2\}, \{X_4, X_1 + X_3\}, \{X_4 + \alpha X_5, X_3\} \\ \{X_4 + 2^{-1} X_5, X_2 + X_3\}, \{X_4 + \alpha X_5, X_2\}, \{X_4 + 2X_5 + \varepsilon X_2, X_1\} \\ \{X_4 + 2X_5 + \varepsilon X_2, X_3\}, \{X_4 - X_5 + \varepsilon X_3, X_1\}, \{X_4 - X_5 + \varepsilon X_3, X_2\} \\ \{X_5 + \varepsilon X_1, X_2\}, \{X_5 + \varepsilon X_1, X_3\}, \{X_4, X_5\} \end{aligned}$$

The three-dimensional subalgebras are

$$\begin{aligned} \{X_1 + \alpha X_3, X_2 + \varepsilon X_3\}, \{X_1 + \varepsilon X_2, X_3\}, \{X_2, X_3\}, \{X_4 + \alpha X_5, X_1\} \\ \{X_4 + X_5, X_1 + X_2\}, \{X_4, X_1 + X_3\}, \{X_4 + \alpha X_5, X_3\} \\ \{X_4 + 2^{-1} X_5, X_2 + X_3\}, \{X_4 + \alpha X_5, X_2\}, \{X_4 + 2X_5 + \varepsilon X_2, X_1\} \\ \{X_4 + 2X_5 + \varepsilon X_2, X_3\}, \{X_4 - X_5 + \varepsilon X_3, X_1\}, \{X_4 - X_5 + \varepsilon X_3, X_2\} \\ \{X_5 + \varepsilon X_1, X_2\}, \{X_5 + \varepsilon X_1, X_3\}, \{X_4, X_5\} \end{aligned}$$

The four-dimensional subalgebras are

$$\{X_4 + \alpha X_5, X_1, X_2, X_3\}, \{X_4, X_5, X_1, X_2\}, \{X_4, X_5, X_1, X_3\}, \{X_4, X_5, X_2, X_3\}$$

2. Invariant solutions

For the one-dimensional subalgebras (besides X_3), there exist invariant solutions. To find them requires that we calculate the invariants of the subalgebra and designate invariants containing functions as new functions of an invariant of the independent variables. The representation of the solution is inserted into system (1.2). In this way, equations are obtained containing only invariants,⁴ i.e., a system of ordinary differential equations. This system is either integrated or reduced to an Abel equation of the second kind (the canonical form of the sub-models).

The first subalgebra (1.3) has a basis of functionally independent invariants

$$s = t - \varepsilon x, \quad \varphi = \delta x, \quad p$$

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