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Stability of the steady rotations of a satellite with internal damping in a central gravitational field[☆]

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ABSTRACT

The rotational motion of a satellite in a central gravitational field in the presence of internal dissipation is studied. The satellite is modelled by a system consisting of two bodies, a shell and a spherical damper. The stability of the steady rotations is investigated for a dynamically symmetric satellite moving in a circular Kepler orbit.

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The problem of the steady rotations of a satellite in a central gravitational field has been most fully investigated for a satellite modelled by a single absolutely rigid body. Within the framework of this model of a dynamically symmetric satellite with a centre of mass that moves in circular Kepler orbit, the existence has been established and the stability investigated of three types of steady rotations: cylindrical, conical and hyperboloidal regular precessions.^{1–4} For a dynamically symmetric satellite with internal dissipation when the satellite is simulated by a rigid body with cavities filled with a viscous fluid, it has been shown^{5–8} that, for a circular orbit, only steady rotations that are cylindrical precessions exist except for equilibria with respect to the orbital basis. The stability of these rotations has been investigated⁸ for quite large values of the viscosity coefficient using the approximate equations proposed by Chernous'ko in which the effect of the viscous fluid on the motion of the supporting body is replaced by a torque calculated from the equations of motion of the “frozen” body.

The steady motions of a satellite, simulated by a system of two bodies, a supporting body (the shell) S_0 and a uniform spheroidal supported body (the damper) S_1 , the centre of which is fixed with respect to the shell (the Lavrent'ev model, Fig. 1), are studied below within the framework of the restricted problem (the motion of the centre of mass and the rotational motion of the satellite are assumed to be mutually independent and the centre of mass moves in a fixed Kepler orbit). The steady motions of a body when there is no external torque,⁹ several properties of the motions of a satellite in a central gravitational field in the unrestricted problem¹⁰ as well as the steady motions of a gyroscope in a uniform gravitational field¹¹ have been investigated earlier within the framework of this model.

1. Equations of motion

The moment of inertia of the damper with respect to its central axis is denoted by I and the central inertial tensor of the whole satellite is denoted by $\mathbf{J}$. Under the assumptions made, the satellite is a gyrostat: its principal central moments of inertia A, B, C are constant. In this case, the expression

$$\mathbf{J}^* = \mathbf{J} - I \mathbf{E}$$

where $\mathbf{E}$ is a unit matrix, is the central inertial tensor of the “auxiliary” body formed by the shell and a point mass equal to the mass of the damper located at the centre of the damper. It is obvious that, regardless of the location of the damper in the supporting body, the

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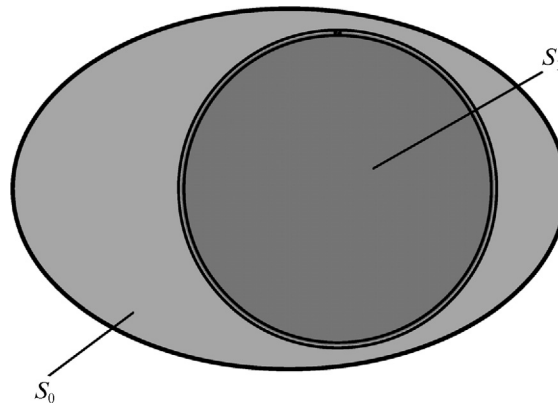


Fig. 1.

positions of the centres of mass and the principal central axes of inertia for the whole satellite and the auxiliary body will coincide. The constraints imposed on the values of the moments of inertia of the satellite:

$$A + B \geq C + I, \quad B + C \geq A + I, \quad C + A \geq B + I \quad (1.1)$$

follow from the triangle inequalities for the auxiliary body.

We denote the absolute angular velocity vector of the shell by ω and the absolute angular velocity vector of the damper by Ω and assume that, in the case of relative displacements of the damper, a dissipative torque acts on it that is proportional to the relative angular velocity of the damper

$$\mathbf{M}_d = -\tilde{\mu} I(\Omega - \omega)$$

Here, the coefficient of proportionality is written in the form of the product $\tilde{\mu} I$, where $\tilde{\mu} > 0$ is a damping coefficient that has the dimension 1/s. This choice of damping coefficient as an independent parameter excludes singularities in the equations of motion of the damper when $I \rightarrow 0$.

The gravitational torque acting on the satellite is determined by the formula³

$$\mathbf{M}_g = 3k \mathbf{R} \times \mathbf{J} \mathbf{R} / R^5 \quad (1.2)$$

where k is a gravitational constant and $\mathbf{R}$ is the radius vector joining the centre of attraction with the centre of mass of the satellite specified by its projections onto the principal central axes of inertia of the satellite.

In the expression for the angular momentum of the satellite with respect to its centre of mass

$$\mathbf{K} = \mathbf{J} \omega + I(\Omega - \omega) \quad (1.3)$$

the first (second) term is the angular momentum of the translational (relative) motion. The angular momentum of the damper with respect to its centre is determined by the expression $\mathbf{G} = I\Omega$. Assuming that the vectors ω and Ω are given by the projections onto the axes of the basis associated with the satellite and using the theorem on the change in angular momentum for the whole satellite and the damper, we obtain the dynamic equations of the system considered

$$\dot{\mathbf{K}} = \mathbf{J} \dot{\omega} + I(\dot{\Omega} - \dot{\omega}) + \omega \times (\mathbf{J} \omega + I \Omega) = \frac{3k}{R^5} \mathbf{R} \times \mathbf{J} \mathbf{R}$$

$$\dot{\mathbf{G}} = I(\dot{\Omega} + \omega \times \Omega) = -\tilde{\mu} I(\Omega - \omega)$$

Subtracting the second equation from the first, we will have

$$(\mathbf{J} - I \mathbf{E}) \dot{\omega} + \omega \times \mathbf{J} \omega = \tilde{\mu} I(\Omega - \omega) + \frac{3k}{R^5} \mathbf{R} \times \mathbf{J} \mathbf{R} \quad (1.4)$$

$$I(\dot{\Omega} + \omega \times \Omega) = -\tilde{\mu} I(\Omega - \omega) \quad (1.5)$$

The first of these equations describes the rotational motion of the auxiliary body and the second describes the rotational motion of the damper.

When $\tilde{\mu} I = 0$ (when there is no damping), Eqs (1.4) and (1.5) are independent and Eq. (1.4) will describe the motion of the auxiliary body under the action of a gravitational torque only, and it follows from Eq. (1.5) that the absolute angular velocity vector of the damper remains constant.

Equations (1.4) and (1.5) are supplemented up to the closed system of kinematic equations of motion of the auxiliary body written in one or another form. For purposes of numerical integration, the most suitable are the Poisson equations in the quaternions

$$2\dot{\Lambda} = \Lambda \circ \omega \quad (1.6)$$

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