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The approach problem of a nonlinear controlled system in a finite time interval☆

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ABSTRACT

A nonlinear controlled system in Euclidean space, which is specified in a finite time interval, is considered. The approach problem of a controlled system with a target set in a finite time interval is studied. An algorithm for an approximate calculation of the solution of the problem is presented. An illustrative example of a "flywheel–pendulum" controlled mechanical system is considered.

The solution of the problem considered of the approach of a nonlinear controlled system with a target set in the phase space of the system can be constructed by isolating the so-called feasibility set (see Refs 1–4), i.e., the set of all initial positions of the controlled system from which this problem is solvable, in the space of positions of the system. Isolation of the feasibility set is a complex and tedious problem; however, its solution provides rich information regarding the possibilities of the controlled system when the target set is reached. The feasibility set can be used to construct programmable admissible controls.[5,6] However, it can be calculated exactly or described effectively by analytical methods only in relatively simple cases; therefore, when approach problems are studied, the problem of the approximate construction of feasibility sets, which is closely related to the approximate design and evaluation of trajectory tubes, integral funnels and reachable sets of dynamical systems, to which the efforts and work of Russian and non-Russian mathematicians have been devoted.[5–13]

Within the scheme described below, the approximate construction of a feasibility set reduces to the approximate construction of feasibility sets for a finite sequence of approach problems of a controlled system with cross sections of the target set that correspond to a finite sequence of instants of time. The correctness of this scheme is established. In addition, a scheme of such a type in the approach problem of a nonlinear stationary controlled system with a "stationary" set in the space of positions of the system, i.e., a set whose time-dependent cross sections are identical, is described. In the last section the approach problem of a stationary controlled "flywheel–pendulum" system with a "stationary" target set in the space of positions of the system is considered. The problem of the transition of a pendulum from the lowest equilibrium position to an upper position was previously studied for a "flywheel–pendulum" system.[14]

1. Approach problem of a controlled system with a non-stationary target set

The controlled system

$$\frac{dx}{dt} = f(t,x,u), \quad x \in R^n, \quad u \in P, \quad P \in \text{comp}(R^r) \tag{1.1}$$

is specified in the time interval $t \in [t_0, T]$, $t_0 < T < \infty$. Here x is the phase vector, and u is the vector of control actions.

System (1.1) satisfies the following conditions.

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Condition 1.1. The vector function $f(t, x, u)$ is defined and is continuous on the set $[t_0, T] \times R^n \times P$, and for any closed bounded region $D \subset [t_0, T] \times R^n$ there is a constant $L = L^D \in (0, x)$, which is such that

$$\|f(t, x_*, u) - f(t, x^*, u)\| \leq L\|x_* - x^*\|; \quad (t, x_*) \text{ and } (t, x^*) \text{ from } D, \quad u \in P \tag{1.2}$$

Condition 1.2. A constant $\gamma \in (0, x)$ exists, which is such that

$$\|f(t, x, u)\| \leq \gamma(1 + \|x\|), \quad (t, x, u) \in [t_0, T] \times R^n \times P \tag{1.3}$$

Condition 1.3. The set

$$F(t, x) = f(t, x, P) = \{f(t, x, u) : u \in P\}$$

is convex for any $(t, x) \in [t_0, T] \times R^n$.

With each closed bounded region $D \in [t_0, T] \times R^n$ we associate the function

$$\omega^*(\delta) = \max\{\|f(t_*, x, u) - f(t^*, x, u)\|: (t_*, x, u) \text{ и } (t^*, x, u) \text{ from } D \times P$$

$$|t_* - t^*| \leq \delta\}, \quad \delta \in (0, \infty) \tag{1.4}$$

The mapping $(t, x) \mapsto F(t, x)$, the constant L^D and the function $\omega^*(\delta)$ satisfy the restrictions

$$d(F(t_*, x_*), \quad F(t^*, x^*)) \leq \omega^*(\delta) + L^D\|x_* - x^*\|, \quad (t_*, x_*) \text{ and } (t^*, x^*) \text{ from } D$$

$$|t_* - t^*| \leq \delta, \quad \omega^*(\delta) \downarrow 0 \text{ when } \delta \downarrow 0 \tag{1.5}$$

$d(F_*, F^*)$ is the Hausdorff distance between the compact sets F_* and F^* in R^n, and the down arrow denotes a monotonic decrease.

We present the definitions of several concepts.

As the admissible control $u(t)$, $t \in [t_0, T]$, we take the Lebesgue measurable vector function $u(t) \in P$, $t \in [t_0, T]$. Let $X(t^*, t_*, x_*) \subset R^n$ ($t_0 \leq t_* < t^* \leq T$, $x_* \in R^n$) be the reachable set of system (1.1) that corresponds to the time t^* and the initial condition $x(t_*)=x_*$; $X(t_*, x_*) = \bigcup_{t* \in [t_*, T]} (t*, X(t*, t_*, x_*))$ is the integral funnel of system (1.1) with the initial position $(t_*, x_*) \in [t_0, T] \times R^n$. Here

$$(t^*, X^*) = \{(t^*, x^*): x^* \in X^*\}, \quad X^* \subset R^n$$

Under conditions 1.1–1.3 the set $X(t^*, t_*, x_*)$ is at the same time the reachable set of the differential inclusion

$$\frac{dx}{dt} \in F(t, x), \quad x(t_*) = x_* \tag{1.6}$$

and is a compact set in R^n. The set $X(t_*, x_*)$ is a compact set in $[t_0, T] \times R^n$ in the case when X_* is a compact set in R^n.

We assume that along with system (1.1) the following target set is specified

$$\mathcal{M} = \{(t, x): t \in [\vartheta, T], x \in \mathcal{M}^t\} = \bigcup_{t \in [\vartheta, T]} (t, \mathcal{M}^t)$$

Here ϑ is a certain instant of time from $[t_0, T]$, and the multiple-valued mapping $t \mapsto \mathcal{M}^t \in \text{comp}(R^n)$ is continuous in $[\vartheta, T]$ in the Hausdorff metric, where $\text{comp}(R^n)$ is the metric space of the compact sets in R^n with the Hausdorff metric $d(\cdot, \cdot)$.

Thus, the target set $\mathcal{M} \subset [\vartheta, T] \times R^n$ has the cross sections $\mathcal{M}^t$, which vary continuously as t varies in $[\vartheta, T]$.

We will formulate the problem of the approach of system (1.1) to $\mathcal{M}$.

Problem 1.1. The requirement is to isolate the set W of all the initial positions (t_*, x_*) of system (1.1) in $[t_0, T] \times R^n$, for each of which an admissible control in $[t_*, T]$ that generates the motion $x(t)$, $x(t_*) = x_*$ of system (1.1) which satisfies the inclusion $(t^*, x(t^*)) \in \mathcal{M}$ at a certain $t^* \in [\vartheta, T]$ can be found.

We will call the set W the feasibility set of Problem 1.1.

Problem 1.1 is next reduced to a set of simpler problems regarding the approach of system (1.1) to the sets $\mathcal{M}^{t^*} \subset R^n$, $t^* \in [\vartheta, T]$ at the times t^*. For this purpose, for each $t^* \in [\vartheta, T]$ we isolate the feasibility set $W^{t^*} \subset [t_0, t^*] \times R^n$ in the problem of the approach of system (1.1) to $\mathcal{M}^{t^*}$ at the time t^* (see, for example, Ref. 6, p. 277). Here W^{t^*} is the set of all the positions $(t_*, x_*) \subset [t_0, t^*] \times R^n$, for each of which an admissible control is found in $[t_0, t^*]$, that transfers the motion $x(t)$, $x(t_*) = x_*$ of system (1.1) at the time t^* into the set $\mathcal{M}^{t^*}$: $x(t^*) \in \mathcal{M}^{t^*}$.

The representation

$$W = \bigcup_{t^* \in [\vartheta, T]} W^{t^*} \tag{1.7}$$

is valid. According to this representation, to calculate the set W, it is necessary to calculate the sets W^{t^*}, $t^* \in [\vartheta, T]$ and to combine them. However, a calculation of all these sets is not realistic, because there is a countless number of such sets. It is more realistic to calculate a certain representative (i.e., a sufficiently numerous) finite collection of a sequence of these sets. This will already be an approximate calculation rather than an exact calculation of the set W.

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