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Instability of a cylinder in the circulation flow of incompressible ideal fluid☆

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ABSTRACT

Within the framework of two-dimensional Euler equations, the stability is investigated of a system consisting of an inner unfastened round cylinder, an incompressible fluid flow around this cylinder with circular streamlines, and an outer fastened cylinder (vessel). An equation is obtained for the natural frequencies with different mean flows between the cylinders. Accurate solutions of this equation are derived and an analysis of these solutions is given. An energetic investigation of the loss of stability in the system is carried out.

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One of the instabilities of three-dimensional vortices, which play a key role in the problem of the onset of turbulence, is the so-called shear instability, related to the transfer of energy to the vortex core from the critical layer arising in the circulation flow as this core oscillates. In particular, instability of this type seems to be responsible for the turbulisation of the vortex core at high Reynolds numbers.[1]

The difficulties encountered when investigating the shear instability in swirling circulation flows are related to the fact that it arises only for curved vortex strands (for example, for a vortex ring), while flow with a cylindrical vortex is stable with any monotonically decreasing vorticity profile. As we know, two-dimensional oscillations of a circular vortex with constant vorticity in an unbounded potential circulation flow are neutrally stable.[2] A circular vortex with constant vorticity likewise retains its stability in a circulation flow with decreasing vorticity.[3] An oscillator in the form of a round cylinder fastened elastically and placed in a potential or swirling circulation flow with decreasing vorticity in an unbounded fluid was examined.[4] It was shown that, in the case of potential flow, the oscillations are always stable, but unlike the oscillations of a cylindrical vortex, the oscillations of a cylinder can prove to be unstable in the case of monotonically decreasing vorticity. This flow seems to be the simplest localised two-dimensional flow in which shear instability can occur.

The mechanism of shear instability, related to the interaction of discrete-spectrum oscillations with perturbations in the critical layer, occurs in plane-parallel flow when surface sea waves interact with the shear flow over the sea surface during wind-driven swell.[5,6] However, in this case, the flow has a different topology compared with the circulation flow about a vortex core. Therefore, it is difficult to extrapolate the results of investigating the emergence of waves on water to processes in a flow containing vortex strands. Furthermore, flows with circular streamlines have an advantage in the mathematical description of effects of this kind, as they allow for obtaining accurate solutions in the case of non-trivial potential flow with a variable velocity over its radius, upon which a weak shear is imposed. In plane-parallel flow, variable velocity is without fail related to strong shear.

Thus, a system consisting of an elastically fastened, rigid, round cylinder in a swirling circulation flow with decreasing vorticity enables an analytical investigation of the shear instability. Unlike the cylindrical vortex oscillations which have a negative energy, the oscillations of a heavy elastic cylinder have a positive energy, and this enables shear instability to be realised in a comparatively simple two-dimensional system.

An experimental investigation of the instability mechanism assumes a bounded flow region. At the same time, shear instability of an elastic cylinder in a circulation flow has been obtained for the case of an unbounded flow.[4] In this connection, the stability problem of oscillations of a cylinder in a vessel of finite radius arises, where the flow is bounded by the walls of the outer stationary vessel and by

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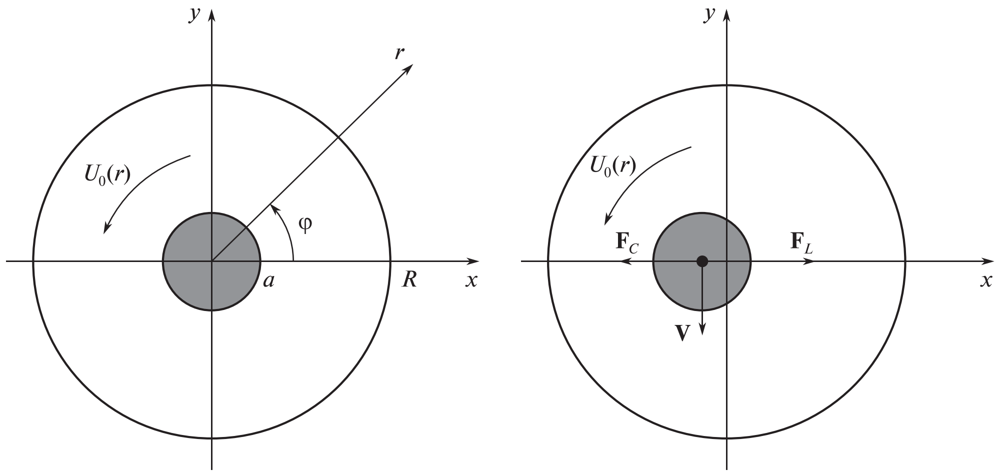

Fig. 1.

the moving round cylinder. The non-triviality of this problem is related to the limitation of the flow region by the vessel wall leading to the onset of one further type of instability, which may mask the shear instability. The problem in this formulation was first considered with a constant and piecewise-constant vorticity between the cylinders.[7] Accurate solutions of the problem were obtained in the case of a constant vorticity between the cylinders or a vorticity decreasing as $1/r$ (Ref. 8).

The aim of the present work is to investigate the instability of a system including a cylinder, a circulation flow, and an outer stationary vessel, and also to analyse the system parameters values for which the shear instability can be separated from the instability related to the boundedness of the system.

The instability mechanisms considered have a non-viscous nature and differ from a viscous instability mechanism, which was investigated both theoretically and experimentally for rapidly rotating rotors in a housing.[9,10]

1. Main equations

In a linear approximation, the stability of an unfastened cylinder of radius a and mass M in a stationary cylindrical vessel of radius R in a potential and swirling incompressible ideal fluid flow is investigated; the fluid density is taken to be equal to unity. The unperturbed state of the system corresponds to a coaxial arrangement of the cylinders (left-hand part in Fig. 1). In a cylindrical system of coordinates r, φ, we set the magnitude of the unperturbed angular velocity of the fluid $U_0(r)$. The stationary fields of vorticity $\Omega_0(r)$ and pressure $P_0(r)$ satisfy the relations

$$\Omega_0(r) = 2U_0(r) + rU_0'(r), \quad P_0'(r) = rU_0^2$$

The position of the cylinder in the perturbed system is described by the displacement vector of its centre from the equilibrium position with the components ξ_x and ξ_y. Then, the following equations will correspond to the natural oscillations of the system:

$$\xi_x = \xi_0 \cos(\omega_R t)\exp(\delta t), \quad \xi_y = \xi_0 \sin(\omega_R t)\exp(\delta t)$$

where ξ_0 is the amplitude of oscillations at the initial instant of time, ω_R is the real part of the natural frequency, and δ is the increment of oscillations, the value of which is non-zero in the instability case; the initial phase of oscillations is taken to be equal to zero. Switching to complex variables $\xi = \xi_x - i\xi_y$, we present the displacement of the cylinder during its oscillations in the form $\xi = \xi_0 \exp(-i\omega t)$, where $\omega = \omega_R + i\delta$ is the complex frequency of oscillation.

To describe the perturbations in the fluid flow, we use an equation for the Lagrangian displacement field $\boldsymbol{\varepsilon}$ related to velocity oscillations by the equation[11,12]

$$\frac{\partial \boldsymbol{\varepsilon}}{\partial t} + \nabla \times (\boldsymbol{\varepsilon} \times \mathbf{U_0}) = \mathbf{v}, \quad \nabla \cdot \varepsilon = 0 \tag{1.1}$$

This approach, based on a description of the perturbed solution in terms of the displacement field, is used to describe the cylinder oscillations in an unbounded fluid.[1,4] The dispacement field is likewise expressed in complex form:

$$\boldsymbol{\varepsilon}(r, \varphi, t) = \boldsymbol{\varepsilon}_0(r)\exp(-i\omega t + i\varphi); \quad \boldsymbol{\varepsilon}_0(r) = \begin{Bmatrix} \varepsilon_r(r) \\ \varepsilon_\varphi(r) \end{Bmatrix} \tag{1.2}$$

Using relations (1.1) and (1.2), we obtain

$$\upsilon_r = (-i\omega + iU_0)\varepsilon_r, \quad \upsilon_\varphi = (-i\omega + iU_0)\varepsilon_\varphi - rU_0'\varepsilon_r \tag{1.3}$$

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