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The motion of a cylinder on a moving plane with friction☆

O.A. Vinogradova

M. V. Lomonosov Moscow State University, Moscow, Russia

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ABSTRACT

The motion of a cylinder on a moving plane with sliding friction and rolling friction is considered. In the case of vertical motion of the plane, it is shown that after a finite time for arbitrary initial conditions one of the following motion modes is established: rest or rolling downwards with or without sliding, accelerated or uniform, depending on the values of the system parameters. In the case of a plane-parallel motion of a horizontal plane, parametric conditions are found for the existence of two periodic rolling modes. It is shown that one of these modes sets up after a finite time for arbitrary initial conditions. An example demonstrating the influence of the torque of rolling friction is considered.

The difference from a similar well-known problem considered earlier[1] is that not only sliding friction is taken into account, but also rolling friction. A qualitative study of the dynamics of the cylinder is performed using the approach and some results formulated by Blekhman and Dzhanelidze.[1] The method of aligning (joining) of solutions of differential equations[2] is applied.

1. Statement of the problem

We consider a dynamically symmetric cylinder of mass m and radius r with moment of inertia I with respect to the axis of symmetry. The cylinder is located on a plane that is inclined at an angle α to the horizontal and is executing translational harmonic oscillations with amplitude A and frequency ω. The direction of the oscillations makes an angle β with the plane.

We introduce a moving coordinate system Oxy bound to the plane, as shown in the Fig. 1. The position of its origin O in the fixed coordinate system $O_1\xi\eta$ with axes parallel to the axes of the Oxy system is given by the law

$$\tilde{\xi} = A\cos\beta\sin\omega t, \quad \tilde{\eta} = A\sin\beta\sin\omega t$$

We assume[1] that

$$-\pi/2 \le \alpha < \pi/2, \quad 0 \le \beta \le \pi/2$$

The cylinder is under the action of the gravity force $m\mathbf{g}$, the normal reaction force $\mathbf{N} = N\mathbf{e}_y$, the friction force $\mathbf{F} = F\mathbf{e}_x$, and the torque of rolling friction $\mathbf{M} = M\mathbf{e}_x \times \mathbf{e}_y$. We write the equations of motion of the cylinder in the moving coordinate system Oxy as follows:

$$m\ddot{x}_s = -m\ddot{\tilde{\xi}} - mg\sin\alpha + F, \quad m\ddot{y}_s = -m\ddot{\tilde{\eta}} - mg\cos\alpha + N, \quad I\ddot{\theta} = Fr + M \tag{1.1}$$

Here x_s and y_s are the coordinates of the centre of mass. We define the friction force according to Coulomb's law:

$$F = \begin{cases} -fN & \text{for} \quad \dot{x}_{gl} > 0 \quad \text{and} \quad \text{for} \quad \dot{x}_{gl} = 0, \quad F^{(0)} \le -fN \\ F^{(0)} & \text{for} \quad \dot{x}_{gl} = 0, \quad \left|F^{(0)}\right| < fN \\ fN & \text{for} \quad \dot{x}_{gl} < 0 \quad \text{and} \quad \text{for} \quad \dot{x}_{gl} = 0, \quad F^{(0)} \ge fN \end{cases} \tag{1.2}$$

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E-mail address: vinogradova-oa@yandex.ru

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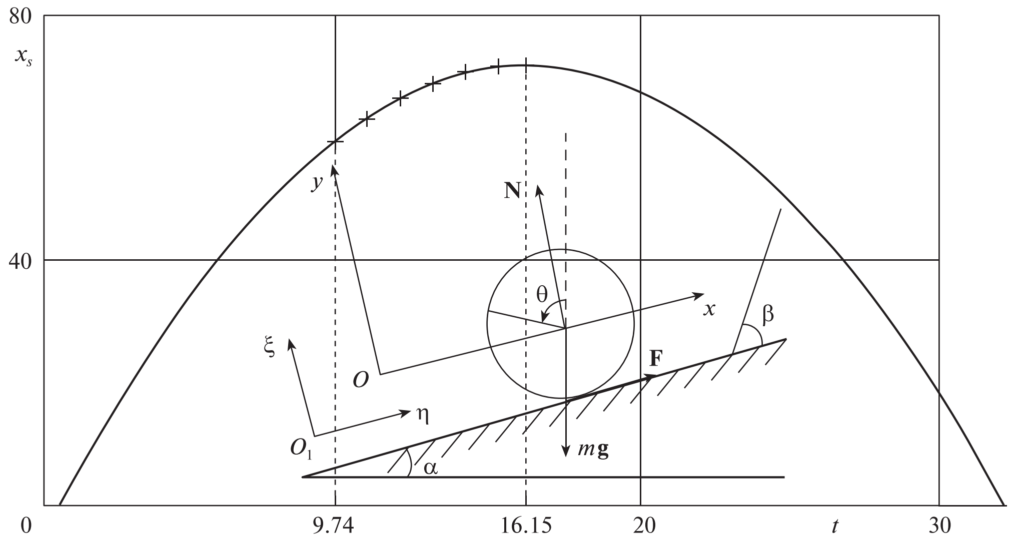

Fig. 1.

Here $\dot{x}_{gl}$ is the velocity of the contact point, $f\,(0<f\leq 1)$ is the coefficient of friction, $F^{(0)}$ is the static friction force. We assign the torque of rolling friction as follows:

$$M = \begin{cases} -\mu r N & \text{for} \quad \dot{\theta} > 0 \quad \text{and} \quad \text{for} \quad \dot{\theta} = 0, \quad M^{(0)} \leq -\mu r N \\ M^{(0)} & \text{for} \quad \dot{\theta} = 0, \quad \left|M^{(0)}\right| < \mu r N \\ \mu r N, & \text{for} \quad \dot{\theta} < 0 \quad \text{and} \quad \text{for} \quad \dot{\theta} = 0, \quad M^{(0)} \geq \mu r N \end{cases} \tag{1.3}$$

Here $\mu > 0$ is the dimensionless coefficient of rolling friction and $M^{(0)}$ is the static torque of rolling friction. On the basis of experimental data[3] we assume that $\mu r \ll f$.

Without loss of generality, we choose the measurement units to be such that $m = 1$, $r = 1$, and $g = |\mathbf{g}| = 1$, and we restrict the discussion to the motion with continuous contact: $y_s = 0$. Thus, proceeding from Eq. (1.1), the magnitude of the normal reaction force is expressed as

$$N = N(t) = \cos\alpha - A\omega^2 \sin\beta \sin\omega t \tag{1.4}$$

The cylinder maintains contact with the plane under the condition of positivity of the normal reaction force. Expression (1.4) allows us to conclude that the motion with continuous contact is possible over the entire period of oscillations of the plane $t \in [0, 2\pi/\omega]$ is the following constraint is imposed on the parameters:

$$\cos\alpha > A\omega^2 \sin\beta \tag{1.5}$$

Note that for the motion with continuous contact Euler's formula is valid: $\dot{x}_s = \dot{x}_{gl} - \dot{\theta}$, taking which into account we write system (1.1) in the form

$$\ddot{x}_{gl} = a(t) + F + \frac{F+M}{I}, \quad \ddot{\theta} = \frac{F+M}{I}; \quad a(t) = -\sin\alpha + A\omega^2 \cos\beta \sin\omega t \tag{1.6}$$

The condition $\mu \ll f$ implies the impossibility of sliding with nonzero velocity without rolling.

Indeed, in the opposite case we obtain from the equations of motion (1.6) an expression for the friction torque $M = -F = fN\mathrm{sgn}\dot{x}_{gl}$, from which the inequality $\left|M\right| > \mu N$ follows that is not in agreement with the chosen law of rolling friction (1.3).

2. Equations of motion

Let us write out the equations of motion of the cylinder using the law of sliding friction (1.2) and the law of rolling friction (1.3). In the case

$$F = -fN \operatorname{sgn} \dot{x}_{gl}, \quad M = -\mu N \operatorname{sgn} \dot{\theta} \tag{2.1}$$

system (1.6) transforms to

$$\begin{aligned} \ddot{x}_{gl} &= a(t) - \frac{(I+1)f \operatorname{sgn} \dot{x}_{gl} + \mu \operatorname{sgn} \dot{\theta}}{I} N(t) \\ \ddot{\theta} &= -\frac{f \operatorname{sgn} \dot{x}_{gl} + \mu \operatorname{sgn} \dot{\theta}}{I} N(t) \end{aligned} \tag{2.2}$$

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