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Modelling the inhomogeneous coating of an elastic plate with optimum sound-reflecting properties☆

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ABSTRACT

The problem of determining the laws governing the inhomogeneity of the coating of a plane elastic plate that ensure the least reflection with a given angle of incidence of a plane sound wave is considered. On the basis of the direct problem solution, a functional representing the intensity of reflection is constructed, and an algorithm for its minimisation is proposed. Analytical expressions describing the mechanical parameters of the inhomogeneous coating are obtained.

The sound-reflecting characteristics of a body can be changed by means of a coating in the form of a continuous inhomogeneous elastic layer. The scattering of sound waves by flat, cylindrical, and spherical elastic bodies with through-thickness inhomogeneous coatings has been investigated.[1–5] A continuous inhomogeneous coating can be modelled by a system of thin homogeneous elastic layers with dissimilar mechanical parameters (density and elastic constants).[6]

The present work is devoted to finding the laws governing the inhomogeneity of the coating of an elastic plate that would ensure the required sound reflection.

The reflection and passage of sound through a plane homogeneous isotropic elastic plate have been investigated in many studies (see, for example, Brekhovskikh[7]). The problem of reflection and refraction of a plane sound wave by an inhomogeneous plane layer has been solved for an isotropic layer,[8] for a transverse isotropic layer,[9] for a layer with anisotropy of the general type,[10] and for a thermoelastic layer.[11]

The recovery of elastic layer characteristics arbitrarily changing over the depth of the layer has been examined.[12] Problems of the recovery of the properties of isotropic and orthotropic through-thickness inhomogeneous layers for a known displacement field at the layer boundary have been solved by analysing steady-state vibrations.[13–15] The linear laws governing the inhomogeneity of a plane elastic layer having least reflection with a given angle of incidence of a plane sound wave have been determined.[16]

1. Statement of the problem

Let us examine a homogeneous isotropic elastic plate of thickness H, the material of which is characterised by a density ρ_0 and elastic constants λ_0 and μ_0. The plate has a coating in the form of a through-thickness inhomogeneous isotropic elastic layer of thickness h (Fig. 1). We assume that the density $\rho(z)$ of the material of the inhomogeneous layer is described by a continuous function, and that the elastic moduli $\lambda(z)$ and $\mu(z)$ are described by differentiable functions of the z coordinate. Here, a system of rectangular coordinates x, y, z is selected such that the x axis lies in the plane separating the plate and the coating, and the z axis is directed downwards along a normal to the plate surface. The external surfaces of the coating and plate are contiguous to ideal homogeneous liquids that have densities ρ_1 and ρ_2 and velocities of sound c_1 and c_2 respectively.

Suppose that, from a half-space $z < -h$, onto a plate with a coating, there falls, at an arbitrary angle, a plane monochromatic sound wave of amplitude A_0, the velocity potential of which

$$\psi_0 = A_0 \exp\{i[k_{1x}x + k_{1z}(z+h) - \omega t]\}; \quad k_{1x} = k_1 \sin\theta_0, \quad k_{1z} = k_1 \cos\theta_0$$

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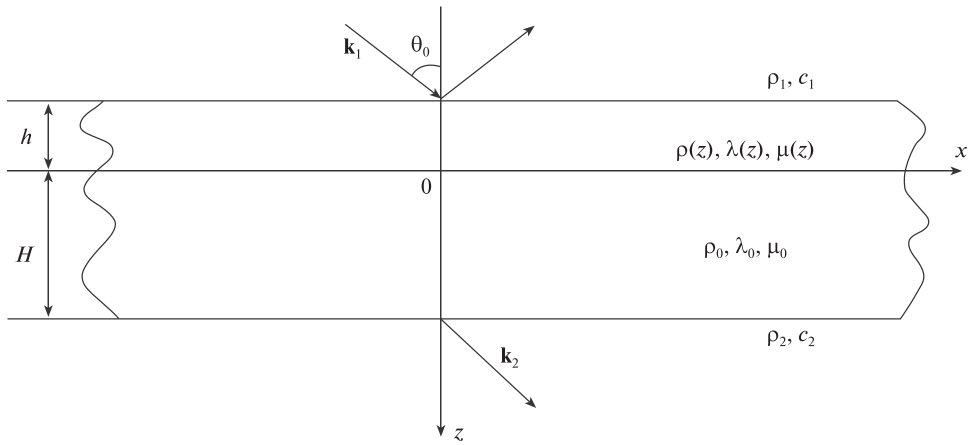

Fig. 1.

where k_{1x} and k_{1z} are projections of the wave vector $\mathbf{k}_1$ onto the x and z axes, $k_1 = \omega/c_1$ is the wave number in half-space $z < -h$, ω is the angular velocity, and θ_0 is the angle of incidence of the plane wave, formed by the normal to the wave front with the z axis. Without limiting the generality, we assume that the wave vector $\mathbf{k}_1$ lies in the xz plane. The time factor $\exp(-i\omega t)$ is omitted below.

The velocity potentials of a wave reflected from a plate with a coating ψ_1 and of a wave passing through it ψ_2 are written in the form[1]

$$\psi_1 = A_1 \exp\{i[k_{1x}x - k_{1z}(z+h)]\}, \quad \psi_2 = A_2 \exp\{i[k_{2x}x + k_{2z}(z-H)]\}$$
$$k_{2x}^2 + k_{2z}^2 = k_2^2$$

where k_{2x} and k_{2z} are the projections of the wave vector $\mathbf{k}_2$ onto the x and z axes, and $k_2 = \omega/c_2$ is the wave number in half-space $z > H$.

The vector of displacement of particles of the elastic homogeneous plate is presented in the form[1]

$$\mathbf{u}^{(0)} = \operatorname{grad}\Psi + \operatorname{rot}\mathbf{\Phi}, \quad \mathbf{\Phi} = \Phi(x,z)\mathbf{e}_y$$

where

$$\Psi = B_1 \exp[i(k_{lx}x + k_{lz}z)] + B_2 \exp[i(k_{lx}x - k_{lz}z)]$$

$$\mathbf{\Phi} = C_1 \exp[i(k_{\tau x}x + k_{\tau z}z)] + C_2 \exp[i(k_{\tau x}x - k_{\tau z}z)]$$

Here, Ψ and $\mathbf{\Phi}$ are the scalar and vector potentials of dispacement, $\mathbf{e}_y$ is the unit vector of the y axis, $k_l = \omega/c_l$ and $k_\tau = \omega/c_\tau$ are the wave numbers of the longitudinal and transverse elastic waves, $c_l = \sqrt{(\lambda_0 + 2\mu_0)/\rho_0}$ and $c_\tau = \sqrt{\mu_0/\rho_0}$ are the velocities of the longitudinal and transverse waves, and k_{lx}, k_{lz}, and $k_{\tau x}$, $k_{\tau z}$ are the projections of the wave vectors of the longitudinal $\mathbf{k}_l$ and transverse $\mathbf{k}_\tau$ waves onto the coordinate axes; $k_{lz}^2 = k_l^2 - k_{lx}^2$, $k_{\tau z}^2 = k_\tau^2 - k_{\tau x}^2$. According to Snell's law,[7] $k_{2x} = k_{lx} = k_{\tau x} = k_{1x}$. The expressions for the coefficients A_j, B_j, C_j ($j = 1, 2$) were given earlier.[1]

The projections of the displacement vector $\mathbf{u}$ onto the coordinate axes in the inhomogeneous coating are written in the form[1]

$$u_x = U_1(z)\exp(ik_{1x}x), \quad u_y = 0, \quad u_z = U_3(z)\exp(ik_{1x}x)$$

The functions $U_1(z)$ and $U_3(z)$ are the solution of the boundary-value problem for a system of linear, homogeneous, ordinary second-order differential equations:

$$A\mathbf{U}'' + B\mathbf{U}' + C\mathbf{U} = 0 \tag{1.1}$$
$$(A\mathbf{U}' + E_1\mathbf{U})_{z=-h} = D, \quad (A\mathbf{U}' + E_2\mathbf{U})_{z=0} = 0 \tag{1.2}$$

where $\mathbf{U} = (U_1, U_3)^T$, and the primes denote differentiation with respect to z. The expressions for the second-order matrices A, B, C, E_1, and E_2 and the column vector D were given earlier.[1] The linear boundary-value problem (1.1), (1.2) was solved by Godunov's orthogonal sweep method.[17]

We will introduce the dimensionless quantities

$$z^* = \frac{z}{h}, \quad \rho^* = \frac{\rho}{\tilde{\rho}}, \quad \lambda^* = \frac{\lambda}{\tilde{\lambda}}, \quad \mu^* = \frac{\mu}{\tilde{\mu}}, \quad A_1^* = \frac{A_1}{A_0}, \quad U_m^* = \frac{U_m}{h}, \quad m = 1, 3$$

where $\tilde{\rho}$, $\tilde{\lambda}$, and $\tilde{\mu}$ are characteristic quantities of the mechanical properties of the coating.

Then, the dimensionless coefficient of reflection is defined by the expression

$$A_1^* = 1 + \frac{\omega h}{A_0 k_{1z}} U_3^*(-1)$$