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The contact problem with the bulk application of intermolecular interaction forces: the influence function for an elastic 'layer–half-space' system☆

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ABSTRACT

The formulation of the contact problem in the presence of bulk forces of intermolecular interaction of contacting bodies is examined. Principal attention is paid to the construction and analysis of the properties of the influence function (Green's function) for an elastic base consisting of a half-space and a layer adhered to it. The behaviour of this function with small and large values of the argument is studied. A numerical analysis of the dependence of the influence function on certain parameters of contact is carried out.

In a study of the contact of deformable bodies, the forces of intermolecular interaction were first taken into account[1] when applied to Hertzian contact. Subsequently, a number of effective approaches to solving contact problems of this type were developed, including approaches using the surface energy concept – the JKR and DMT models.[2,3] Fairly comprehensive reviews are available[4,5] of work on calculating the contact of bodies in the presence of intermolecular interaction, including for bodies with coatings.

The strict formulation of the contact problem in the presence of forces of intermolecular interaction presupposes the existence of a certain gap between the contacting bodies. The size of this gap depends on the strain of the bodies and should ensure equality of deformation and intermolecular forces on the contact (Derjaguin's self-consistent approach). Formulation of the contact problem within the framework of a self-consistent approach was examined for the first time for a spherical Hertzian contact in the presence of intermolecular interaction forces described by the Lennard-Jones potential.[6] This approach has been further developed in a number of studies,[7–10] and it has been customary to assume that the intermolecular interaction forces are applied to the boundary of the body as a contact pressure.

Allowance for bulk forces of intermolecular interaction was made earlier within the framework of the general formulation of problems of the mechanics of deformable bodies,[11,12] and also in analytical analysis of the stress–strain state of certain elastic bodies of canonical shape.[13]

1. Formulation of the problem and the principal equations

These will be formulated for an inhomogeneous elastic base with arbitrary change in physicomechanical properties over its depth. On contact with such a base of an absolutely rigid body (indentor), the presence of intermolecular interaction forces leads to the existence of a certain contact gap r (Fig. 1a) ensuring a balance of the forces of intermolecular and elastic interaction of the contacting bodies.[6] A well-known approach[9,14,15] based on the summation of paired interactions of molecules (Hamaker's hypothesis) makes it possible to calculate the force p of action of the indentor on the base per unit area of its boundary:[13]

$$p = \Phi(r) \equiv -\int_0^\infty f(\lambda, r)d\lambda \tag{1.1}$$

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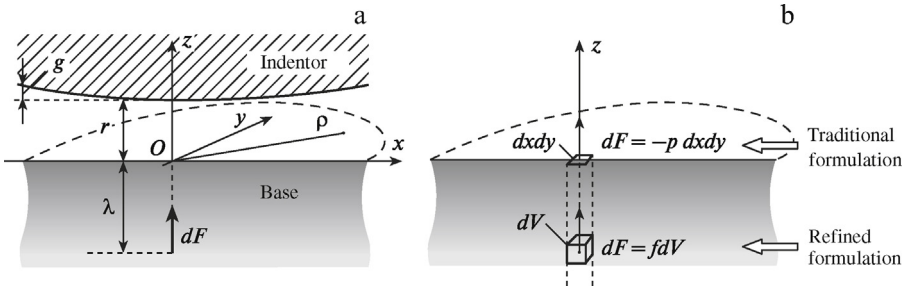

Fig. 1.

where $f(\lambda, r) = N(\lambda) f_s(\lambda, r)$ is the bulk force in the base, $f_s(\lambda, r)$ is the force acting from all the molecules of the indentor on a certain molecule of the base positioned at a distance λ from its boundary, and $N(\lambda)$ is the concentration of molecules of the base. Definition (1.1) of quantity p enables it to be interpreted as a contact pressure.

Remarks.

1. The forces of paired interaction of molecules, the summation of which determines the force f_s, depend on the properties of the molecules and the distance between them. Verious forms of this dependence exist, among which the most well known is the dependence defined by the Lennard-Jones potential.[16] A distinguishing feature of the forces of paired interaction of molecules is their rapid decay at distances exceeding a certain characteristic size r_e.
2. On each molecule of the base there act, besides molecules of the indentor, the other molecules of the base. This action leads to the appearance in the base of certain preliminary stresses and strains,[12,17] which have no influence on the solution of the contact problem within the framework of the linear theory of elasticity.
3. The contact pressure $p = \Phi(r)$ can be calculated by the above procedure, but assuming the opposite, that f_s is a force acting on a certain molecule of the indentor from all the molecules of the base, and N is the concentration of molecules of the indentor, which may also be inhomogeneous over its depth. It is simple to verify that, in such calculation, the former dependence $\Phi(r)$ is obtained.

Note that the theory set out here presupposes that the gap r changes weakly within regions of size $\sim r_e$ in the contact area. Furthermore, the extent of the contact area is assumed to be much greater than the gap r and the quantity r_e. These assumptions enable the procedure of summation of the paired interactions of molecules to be simplified, assuming that the gap r is locally constant and replacing the volume of the indentor with a half-space. In particular, with such assumptions, relation (1.1) is derived.

The traditional formulation of the contact problem in the presence of intermolecular interaction implies that the contact pressure determined by means of formula (1.1) is applied to the base, boundary as a result of which the base is deformed.[6–10] Below, principal attention is paid to a refined formulation in which it is naturally assumed that the strain of the base is generated by bulk forces f distributed over its depth in a known way, whereas the base boundary is free of loads.

This can be expressed mathematically in the following way. Suppose that, for the base considered, the solution of Mindlin's problem of the action of an internal concentrated force is known.[18–21] In particular, this solution makes it possible to determine the normal displacement dw of the base boundary by the action of a vertical force dF applied to the base at depth λ:

$$dw(\rho) = M(\rho, \lambda) dF \tag{1.2}$$

where M is a known function, $\rho = \sqrt{x^2 + y^2}$ is a radial coordinate, and the system of coordinates $Oxyz$ is connected with the boundary of the undeformed base such that the x and y axes are located on this boundary (Fig. 1a).

With the traditional formulation of the problem, in relation (1.2) it is necessary to assume that $dF = -p dx dy$, $\lambda = 0$ (the Boussinesq problem,[22] Fig. 1b), whereas with the refined formulation it is assumed that $dF = f(\lambda, r) dV$, $\lambda \in [0, \infty)$, and summation of the contributions to the displacement dw from all elementary volumes $dV = dx dy d\lambda$ positioned beneath the boundary element $dx dy$ is carried out.[13] As a result of the indicated actions, the equation

$$dw(\rho) = \Gamma(\rho; r) dx dy \tag{1.3}$$

is obtained, and here

$$\Gamma(\rho; r) = \int_0^\infty f(\lambda, r) M(\rho, \lambda) d\lambda - \text{refined formulation} \tag{1.4}$$

$$\Gamma(\rho; r) = -M(\rho, 0) \Phi(r) - \text{traditional formulation} \tag{1.5}$$

The quantity Γ in Eq. (1.3) can be interpreted as the influence function (Green's function) determining the normal displacement of the base boundary from local action of the indentor on the base material located beneath the element $dx dy$, and here the gap r serves as a characteristic of the external load.

In the case of an arbitrary distribution $r(x, y)$ of the gap over the contact area $\mathscr{D}$, following a known procedure (see, for example, Johnson[23]), we will integrate the results of such local effects [i.e., the right-hand side of Eq. (1.3)] over the area $\mathscr{D}$ and obtain for the displacement of the base boundary the following expression:

$$w(x, y) = \iint_{\mathscr{D}} \Gamma\left(\sqrt{(\xi - x)^2 + (\eta - y)^2}; r(\xi, \eta)\right) d\xi d\eta \tag{1.6}$$

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