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The effect of surface stresses on the stress–strain state of shells☆

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ABSTRACT

The effect of surface stresses on the stress–strain state (SSS) of an elastic shell is investigated. The surface stresses are represented as a static pre-loading localized in ultra-thin layers of the shell close to its surface. It is assumed that the mechanical properties of the surface layers differ from the properties of the material far from the surface. The three-dimensional elasticity equations are analysed by an asymptotic method using several asymptotic parameters. Equations are obtained by the method of reducing the three-dimensional equations of the theory of elasticity to the two-dimensional equations of shell theory in which surface stresses have to be taken into account for a sufficiently small shell thickness. Asymptotic estimates of the effect of surface stresses on the SSS are obtained as a function of the ratio of the elastic moduli of the shell material and of the layer close to the surface, the ratio of the shell thickness to the thickness of the surface layer and the type of SSS and its variability with respect to the coordinates.

The study of the effect of surface stresses on the stress–strain state (SSS) of rigid bodies has already been started by Laplace[1] and Young.[2] Reviews of papers on this topics are available (see Refs 3–5, for example). This problem has become pressing in connection with the active use of elements, the dimensions of which are commensurate with molecular dimensions.[6,7]

Surface stresses arise as a consequence of the interaction of molecules on the surface of a rigid body not only with the neighbouring molecules of the body but also with the molecules of the surrounding medium ((solid, liquid, gaseous) or with a vacuum. The surface stresses of many liquids are measured as a function of temperature. In the case of rigid bodies, these measurements are associated with great difficulties, and different methods are available for their realization.[8–11] For example, the Gibbs–Curie–Wolf equation

$$\frac{\sigma_1}{h_1} = \frac{\sigma_2}{h_2} = \ldots = \frac{\sigma_i}{h_i}$$

is used for the equilibrium form of crystals, where σ_i are the surface stresses of the i-th edge of the crystal and h_i is its distance from the centre of the crystal.

The magnitude of the ideal surface stresses of a rigid body σ_T can be found from the Polanyi, Smekal and Griffith equation

$$P_{\text{id}} = (2\sigma_T E/\delta)^{1/2}$$

where P_{id} is the ideal value of the ultimate strength stress of the body and δ is the intermolecular (interatomic) distance.

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1. Surface stresses in shells

We will assume that surface stresses are the forces due to intermolecular interactions close to the body surface per unit length of the contour bounding an element of a thin layer near the surface. The surface stresses are perpendicular to the bounding contour and have the dimension Nm^{-1}.

If the smallest characteristic dimension of a deformed body is considerably greater than the thickness of the layer in which the surface stresses act, the effect of surface stresses on the SSS of the body is unimportant and it can be neglected.

The results of a number of papers (see Refs 12 – 14) confirm that the surface stresses can turn out to be considerable even if the body has just one small characteristic dimension (for example, thin bodies such as thin filaments, plates, shells, bodies with cracks or small apertures, and so on).

We will treat surface stresses as a static pre-loading localized in ultrathin layers near the faces of a shell. The effect of static preloading on the SSS of structures has been thoroughly studied and described in a number of papers (see Refs 15 and 16, for example). The surface stresses must enter into the initial three-dimensional equations of the theory of elasticity since they are always present near the body surface.

We will analyse the role of surface stresses in the SSS of an elastic body using the example of a shell which enables us to study the dependence of the effect of surface stresses on the SSS as a function of a number of parameters, that is, the shell thickness, the thickness of the layer in which the surface stresses act, the physical properties of the material, the variability of the SSS and the type of SSS.

As previously,[12] we represent a uniform shell in the form of a three-layer shell of symmetric structure (R is its characteristic dimension) consisting of a central layer of thickness $2h_0$ and two ultrathin layers of thickness h_1, adjoining the faces of the shell, in which the surface stress forces σ_0 with the dimension Nm^{-1} are localized. We represent the surface stress forces as stresses $\tau_0 = \sigma_0/h_1 = \text{cont.}$ that are uniformly distributed over the thickness of the ultrathin layer.

It should be noted that the results obtained below are estimates: the surface stresses τ_0 are not at all constant throughout the surface layer and the surface layer material is not isotropic as assumed here. It can be either transversally anisotropic or be described by other models differing from the classical linear theory of elasticity. For example, the theory of micropolar shells has been used in describing surface stresses.[17]

The asymptotic method enables us to investigate any problems having small parameters but, since there is no reliable information concerning the equations of state of a surface layer, we will use the classical theory of elasticity.

We write the three-dimensional equations of the theory of elasticity for ultrathin layers in which there are static loads acting in the tri-orthogonal curvilinear system of coordinates $\alpha_1, \alpha_2, \gamma$ (α_1 and α_2 are the lines of curvature of the middle surface of the shell and the γ axis is orthogonal to them):

the equilibrium equations

$$\frac{1}{A_i}\frac{\partial \overset{k}{\tau}_{ii}}{\partial\alpha_i} + \frac{1}{A_j}\frac{\partial \overset{k}{\tau}_{ij}}{\partial\alpha_j} + k_j\left(\overset{k}{\tau}_{ii} - \overset{k}{\tau}_{jj}\right) + k_i\left(\overset{k}{\tau}_{ij} + \overset{k}{\tau}_{ji}\right) + \frac{1}{a_i}\frac{\partial}{\partial\gamma}\left(a_i^2\overset{k}{\tau}_{i3}\right)$$
$$- a_i a_j \tau_0\left(\frac{1}{A_j}\frac{\partial \overset{k}{m}_{ij}}{\partial\alpha_j} + k_j\left(\overset{k}{e}_i - \overset{k}{e}_j\right)\right) = 0 \qquad (1.1)$$

$$-\frac{\overset{k}{\tau}_{11}}{R_1} - \frac{\overset{k}{\tau}_{22}}{R_2} + \frac{1}{A_1}\frac{\partial \overset{k}{\tau}_{13}}{\partial\alpha_1} + \frac{1}{A_2}\frac{\partial \overset{k}{\tau}_{23}}{\partial\alpha_2} + k_2\overset{k}{\tau}_{13} + k_1\overset{k}{\tau}_{23} + \frac{\partial \overset{k}{\tau}_{33}}{\partial\gamma}$$
$$+ a_1 a_2 \tau_0\left(\frac{1}{A_1}\frac{\partial \overset{k}{g}_1}{\partial\alpha_1} + \frac{1}{A_2}\frac{\partial \overset{k}{g}_2}{\partial\alpha_2} + k_1\overset{k}{g}_2 + k_2\overset{k}{g}_1\right) = 0 \qquad (1.2)$$

the equations of state

$$\frac{\partial \overset{k}{\upsilon}_3}{\partial\gamma} = \frac{1}{E_k}\frac{1}{a_1a_2}\overset{k}{\tau}_3 - \frac{\nu_k}{E_k}\frac{1}{a_2}\overset{k}{\tau}_1 - \frac{\nu_k}{E_k}\frac{1}{a_1}\overset{k}{\tau}_2 - 2\nu_1\frac{\tau_0}{E_1}, \quad \frac{\partial \overset{k}{\upsilon}_i}{\partial\gamma} = -\frac{1}{a_i}\overset{k}{g}_i + \frac{2(1+\nu_k)}{E_k}\frac{1}{a_j}\overset{k}{\tau}_{i3}$$
$$\overset{k}{\tau}_i = -\frac{E_k}{1-\nu_k^2}\left(\frac{a_j}{a_i}\overset{k}{e}_i + \nu_k\overset{k}{e}_j\right) + \frac{\nu_k}{1-\nu_k}\frac{1}{a_i}\overset{k}{\tau}_3 - \frac{a_j}{1-\nu_1}\tau_0, \quad \overset{k}{\tau}_{ij} = \frac{E_k}{2(1+\nu_k)}\left(\frac{a_i}{a_j}\overset{k}{m}_i + \overset{k}{m}_j\right) \qquad (1.3)$$

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