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Non-linear limit cycle flutter of a plate with Hertzian contact in axial flow

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HIGHLIGHTS

- A complete dynamical model of a plate with Hertzian contact and in an axial flow is developed.
- The reactive fluid force is solved by the Glauert's method, and the viscous effect is evaluated as a resistive fluid force.
- A heuristic analysis scheme constructed upon the equivalent linearized method is applied for nonlinear limit cycles flutter.
- The system experiences flutter instability, limit cycles flutter, lock-in motions, quasi-periodic motions and quasi-periodic divergence.

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ABSTRACT

This paper is aimed at the nonlinear flutter of a cantilevered plate with Hertzian contact in axial flow. The contact effect is modeled as a nonlinear spring force with both square and cubic nonlinearity. The fluid force is considered as the sum of two parts, one is the reactive fluid force due to plate motion and the other is the resistive fluid force independent on plate motion. The reactive fluid force is derived by solving the bound and wake vorticity with the help of Glauert's expansions, and the resistive force is evaluated in terms of drag coefficient. The governing nonlinear partial differential equation of the system is discretized in space and time domains by using the Galerkin method. Results show that the plate loses its stability by flutter and then undergoes limit cycle motions due to the contact nonlinearity after instability. The present fluid model is reliable and shows good agreement with other theories archived. A heuristic analysis scheme based on the equivalent linearization method is developed for the analysis of bifurcations and limit cycles. The Hopf bifurcation is either supercritical or subcritical, which is closely dependent on the contact location. For some special cases the bifurcations are, interestingly, both supercritical and subcritical. When the plate experiences limit cycles, with the increasing dynamic pressure there firstly appear the lock-in motions; and then the quasi-periodic motions show up as a breaking of limit cycle by inclusion of a secondary significant frequency with an irrational value of $\frac{1}{4\pi}$ of the dominant limit cycle frequency. Finally the plate undergoes dynamic buckling characterized by quasi-periodic divergence when the dynamic pressure is relatively large.

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1. Introduction

Plates immersed in uniform axial flow can be found many applications in science and engineering, such as for aircraft and missile skins (Dowell, 1975, 1967) in supersonic flow, and for paper-making (Watanabe et al., 2002b, a), electricity generation (Allen and Smits, 2001) and aircraft control (Breuker et al., 2008) in low-speed flow. An early excellent monograph on this problem was published by Dowell (1975) and a recent review was presented in the book by Païdoussis et al. (2010).

More recently, this problem has received a renewed application for high-speed trains. It is well known that the record of the maximum train speed during test is being bettered constantly in recent decades. However, high-speed running conditions result in some unavoidable aerodynamic problems (Raghunathan et al., 2002; Zhu et al., 2014). Such problems are receiving more and more attention as a practical engineering issue requiring urgent resolutions. And the optimization of aerodynamic performance has been one of the main objective of the high-speed trains design for low energy consumption, high safety and stability requirements (Raghunathan et al., 2002; Ding et al., 2016). The high-speed trains generally adopt the streamlined design to decrease running resistance, thus thin plate structures such as the train body skin are widely used. As shown in Fig. 1, the train skin composed of many pieces of plates is welded onto the train keels. These plates traveling at high speed through air can be easily excited by fluid enveloping them. For instance, in the test of Wuhan–Guangzhou railway passenger dedicated line of China, severe vibrations of the train-body skin and strong noise radiation are observed when the CRH-3 trains moved at almost 450 km/h (a test speed higher than the practical commercial running speed of 300 km/h).

In fact the above problems involve the studies of both fluid and plate mechanics fundamentally, and are the typical aero-elastic (or fluid–structure interaction) problems. The main feature of such problems is that the plate responses also modify the fluid forces in a feedback sense when such fluid forces excite the plate. A plate vibrating in flow may experience instability. The type of instability, either static (divergence) or dynamic (flutter) is closely dependent on both the plate boundary conditions and flow velocity. For instance, a plate fixed at its both ends (simply supported or clamped) undergoes divergence in low speed flow (Kornecki et al., 1974; Li et al., 2012) but flutter instability in supersonic flow (Dowell, 1975, 1967); conversely, if a plate is cantilevered at one end and free at the other, it flutters in low speed flow but undergoes divergence in supersonic flow (Kornecki et al., 1974). Roughly speaking, the instability type is mainly dependent on the contribution of the fluid force to the plate. From the previous results (Dowell, 1975, 1967; Kornecki et al., 1974; Li et al., 2012), it follows that the fluid force is proportional to the plate's slope in supersonic flow but to the plate's curvature in low speed flow.

On the aero-elastic instability of a plate in flow, relevant researches are generally carried out within the framework of linear modeling and theory. Despite its simplicity, these models can describe the most important nature of plate instability. Although the plate is almost modeled as the same linear beam, the results are totally different if the fluid force is calculated by different theories. A simply supported plate was reported to flutter when it is exposed to an airflow of Mach number of 0.125 by Dugundji et al. (1963), and this result was also verified by their wind tunnel test. In Dugundji et al. (1963), the fluid force was considered to be that on an infinite long wavy (sinusoidal) wall, and was obtained by the plate modes expansion. However, Ishii (1965) shown that such a simply supported plate loses its stability by divergence; in this work, the flow velocity potential was expressed by a time varying sources distribution on the plate, and the fluid force was obtained by the standard mode method. The results by Ishii (1965) are in agreement with those by Kornecki et al. (1974), who applied the Theodorsen's theory for the fluid force. In fact, flutter of plates with both ends fixed (simply supported or clamped) has not been reported either as theory or experiment in the relevant literatures as explored by the authors (Li et al., 2012; Ellen, 1977; Guo and Paidoussis, 2000).

As previously noted that if the fluid is modeled by different theories, a plate fixed at its both ends may undergo different type instability; however, as for a cantilevered plate there is almost a consensus on its flutter instability (Kornecki et al., 1974; Zhang et al., 2000; Datta and Gottenberg, 1975; Huang, 1995; Balint and Lucey, 2005; Yamaguchi et al., 2000; Tang et al., 2009; Tang and Paidoussis, 2007; Tang et al., 2003; Shayo, 1980). The main purpose of relevant studies on plate flutter is to give a theoretical prediction of the critical flow velocity and frequency in terms of plate parameters. Datta and Gottenberg (1975) examined this problem with the fluid force obtained from the slender wing theory. Huang (1995), Kornecki et al. (1974) and Shayo (1980) used the Theodorsen's theory for the fluid force to study plate flutter. Balint and Lucey (2005), on the other hand, applied a Navier–Stokes solver for the fluid force. Yamaguchi et al. (2000) applied a linear time variable vortex model considering the shedding wake for fluid, and modeled the plate as a lifting surface. Watanabe et al. (2002b) used both the Theodorsen's theory and Navier–Stokes solver to form the fluid force. A two-dimensional and a three-dimensional vortex lattice methods for plate flutter are respectively reported by Tang et al. (2009), Tang and Paidoussis (2007) and Tang et al. (2003). Li et al. (2015); Li and Yang (2014) modeled the flow force by using a time-dependent source on the plate and considered the plate as a lifting surface. Moreover, a series of experiments on the flutter mechanism have been conducted by Zhang et al. (2000) and Shelley et al. (2005) in water flow and by Kornecki et al. (1974), Datta and Gottenberg (1975), Tang et al. (2009), Tang and Paidoussis (2007) and Watanabe et al. (2002a) in wind tunnel. These experiments corroborate the stability criteria predicted by theoretical analysis: the stabilizing effect of decreasing flow velocity, decreasing plate length, decreasing plate mass and increasing plate bending rigidity.

Another interesting aspect of plate aero-elastic problems is the post-instability dynamics due to nonlinearity. Such nonlinearity generally comes from both the plate structure and the flow. One common type structural nonlinearity arises from the large amplitude vibration of the plate. The limit cycle flutter of a cantilevered plate in axial flow was examined