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Modal decomposition of fluid–structure interaction with application to flag flapping

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ABSTRACT

Modal decompositions such as proper orthogonal decomposition (POD), dynamic mode decomposition (DMD) and their variants are regularly used to educe physical mechanisms of nonlinear flow phenomena that cannot be easily understood through direct inspection. In fluid–structure interaction (FSI) systems, fluid motion is coupled to vibration and/or deformation of an immersed structure. Despite this coupling, data analysis is often performed using only fluid or structure variables, rather than incorporating both. This approach does not provide information about the manner in which fluid and structure modes are correlated. We present a framework for performing POD and DMD where the fluid and structure are treated together. As part of this framework, we introduce a physically meaningful norm for FSI systems. We first use this combined fluid–structure formulation to identify correlated flow features and structural motions in limit-cycle flag flapping. We then investigate the transition from limit-cycle flapping to chaotic flapping, which can be initiated by increasing the flag mass. Our modal decomposition reveals that at the onset of chaos, the dominant flapping motion increases in amplitude and leads to a bluff-body wake instability. This new bluff-body mode interacts triadically with the dominant flapping motion to produce flapping at the non-integer harmonic frequencies previously reported by Connell and Yue (2007). While our formulation is presented for POD and DMD, there are natural extensions to other data-analysis techniques.

1. Introduction

Modal decompositions such as proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD) have been used to distill important physical mechanisms from data and to develop reduced-order models for turbulent wall-bounded flows (Berkooz et al., 1993), flow past a cylinder (Chen et al., 2012; Bagheri, 2013), and a jet in cross-flow (Rowley et al., 2009; Schmid, 2010), to name a few examples.

These techniques were developed for flows involving (at most) stationary immersed surfaces, and have been applied less extensively to fluid–structure interaction (FSI) problems, where the fluid motion is coupled to deformation and/or vibration of an immersed structure. In this FSI setting, data analysis has, to our knowledge, only been applied to data of either the fluid or the structure independently of the other. The fluid-only approach has been used to study flow past a flexible membrane (Schmid, 2010), a cantilevered beam (Cesur et al., 2014), and an elastically-mounted cylinder undergoing vortex-induced vibration (Blanchard et al., 2017). The solid-only approach has been applied to fish swimming (Bozkurttas et al., 2009;

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Tangorra et al., 2010) and flag flapping (Michelin et al., 2008; Kim et al., 2013). These approaches reveal significant flow or structure behavior, respectively, but do not yield driving mechanisms in the omitted quantity. This in turn leaves the correlation between fluid and structure behavior unknown.

We propose a framework for data analysis of FSI systems where the fluid and structure are treated together, which naturally allows correlation between the fluid and structure to inform the resulting modes of the fully-coupled system. As part of this formulation, we define a norm in terms of the total mechanical energy of the FSI system. This combined fluid–structure data-analysis procedure is then demonstrated on limit-cycle flapping and chaotic flapping of strictly two-dimensional flags.

In the case of chaotic flapping, previous work identified that for flags of low stiffness, chaos can be triggered by increasing the flag mass (Connell and Yue, 2007; Alben and Shelley, 2008). For flows at moderate Reynolds numbers of $O(1000)$, Connell and Yue (2007) showed that the flag system transitions with increasing mass from a stable equilibrium to limit-cycle flapping of increasing amplitude, then to chaotic flapping. Alben and Shelley (2008) found similar transitions in inviscid fluids. In the viscous case, the transition to chaos is associated with the appearance of a distinct frequency that is a noninteger harmonic of the dominant flapping frequency (Connell and Yue, 2007), though the cause of this new frequency signature is as yet unexplained. We use our coupled FSI decomposition to identify the mechanism responsible for the appearance of this noninteger frequency harmonic.

We focus here on proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD) because of their widespread use and their expected suitability for the problems considered here. The limit-cycle case described in Section 3.1 is associated with one dominant frequency, and thus DMD is a natural candidate because of its localized harmonic nature (Mezić, 2013). POD is also expected to be suitable because of the near-harmonic decomposition it typically yields for limit-cycle flows (such as occurs in vortex shedding past a cylinder near the critical Reynolds number of approximately 47; see, *e.g.*, Kutz et al., 2016). For the chaotic flapping problem described in Section 3.2, the non-broadband ('peaky') nature of the dynamics again makes DMD a fitting technique. However, POD and DMD are not ideal for all contexts. For example, Towne et al. (2018) demonstrated that in statistically stationary flows with broadband frequency content — as observed in the majority of turbulent flows — spectral POD provides an optimal decomposition. The major goal of the current work is to demonstrate the utility of performing data analysis in a manner that accounts for both the fluid and the structure, rather than explore the advantages of any particular technique, a question which in any event depends on the specific FSI problem under consideration. Future work can readily incorporate the methodology presented here into the appropriate technique for the intended application.

2. POD and DMD of fluid–structure interaction

We consider snapshot-based methods applied to discrete data. The associated data matrices are assumed to be organized so that each column provides the state of the system at an instance in time and each row contains the time history of a specific state variable. For simplicity, the formulation is presented in a two-dimensional setting; the extension to three dimensions is straightforward.

We assume fluid data is given on a stationary Cartesian grid, Ω, made up of n_f points ($\Omega \subset \mathbb{R}^{1\times n_f}$), and let the streamwise and transverse fluid velocities at the ith time instance, t_i, be $\mathbf{u}_i, \mathbf{v}_i \in \Omega$. Fluid data is often provided in this format by immersed-boundary methods and experiments; some numerical methods use moving meshes at each time step that conform to the moving structure, and fluid data obtained from these methods would need to be interpolated onto a single stationary grid at each time instance to use the method we propose here.

We consider structural data provided in a Lagrangian setting, with the structural domain, Γ, comprised of n_s points ($\Gamma \subset \mathbb{R}^{1\times n_s}$ depends on time). We let $\boldsymbol{\chi}_i, \boldsymbol{\eta}_i \in \Gamma$ denote the streamwise and transverse structural displacements from an undeformed reference configuration at the ith time instance, and $\boldsymbol{\xi}_i, \boldsymbol{\zeta}_i \in \Gamma$ be the corresponding structural velocities. We define the total state vector at t_i as $\mathbf{y}_i = [\mathbf{u}_i, \mathbf{v}_i, \boldsymbol{\chi}_i, \boldsymbol{\eta}_i, \boldsymbol{\xi}_i, \boldsymbol{\zeta}_i]^T \in \mathbb{R}^{2n_f+4n_s}$, and define the data matrix, $\mathbf{Y} \in \mathbb{R}^{n\times m}$ ($n = 2n_f + 4n_s$ is the size of the state and m is the number of snapshots), as $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_m]$.

POD modes are computed from the mean-subtracted data matrix, $\tilde{\mathbf{Y}}$, whose ith column is defined as $\tilde{\mathbf{Y}}_i = \mathbf{Y}_i - \boldsymbol{\mu}$, where $\boldsymbol{\mu} = 1/m\sum_{k=1}^{m}\mathbf{y}_k$ is the sample temporal mean of $\mathbf{Y}$. For DMD, Chen et al. (2012) found that the use of $\tilde{\mathbf{Y}}$ reduces DMD to a discrete Fourier transform in time, and that using $\mathbf{Y}$ allows for growth-rate information to be retained. For this reason, DMD is performed on $\mathbf{Y}$ below.

The immersed structure is assumed to be thin in the ensuing discussion, as occurs for problems involving flags, bio-inspired wings and fins, spring-mounted flat plates, *etc*. For bodies of non-negligible thickness, points on Ω lie within the immersed body at any time instance, leading to spurious contributions from the 'fictitious fluid' within $\Omega \cap \Gamma$. Addressing this challenge is a subject of future work.

2.1. Proper orthogonal decomposition

POD decomposes the data into orthogonal spatially uncorrelated modes that are ordered such that the leading k modes ($k \leq m$) provide the most energetically dominant rank-k representation of $\tilde{\mathbf{Y}}$. This optimal representation is defined with respect to a norm, and we therefore select an inner product space whose induced norm yields the mechanical energy of the FSI system. We first motivate this choice of norm within a continuous-variable setting, and subsequently provide details