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Time-domain and modal response of ice shelves to wave forcing using the finite element method

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ABSTRACT

The frequency-domain and time-domain response of a floating ice shelf to wave forcing are calculated using the finite element method. The boundary conditions at the front of the ice shelf, coupling it to the surrounding fluid, are written as a special non-local linear operator with forcing. This operator allows the computational domain to be restricted to the water cavity beneath the ice shelf. The ice shelf motion is expanded using the in vacuo elastic modes and the method of added mass and damping, commonly used in the hydroelasticity of ships, is employed. The ice shelf is assumed to be of constant thickness while the fluid domain is allowed to vary. The analysis is extended from the frequency domain to the time domain, and the resonant behaviour of the system is studied. It is shown that shelf submergence affects the resonant vibration frequency, whereas the corresponding mode shapes are insensitive to the submergence in constant depth. Further, the modes are shown to have a property of increasing node number with increasing frequency.

1. Introduction

We are interested in modelling the impact of very long ocean surface waves on ice shelves, primarily waves in the tsunami–infragravity regime. The phenomenon has been the subject of recent observations and measurements (Cathles et al., 2009; Bromirski et al., 2015, 2017), which have shown that there is substantial coupling between ocean waves and ice shelf vibrations. MacAyeal et al. (2006) provide evidence of a connection between wave-induced vibrations and shelf calving and possible collapse, although the strength of the connection remains unknown.

In the present work, we describe bespoke code to calculate the time-dependent response of an ice shelf to wave forcing, using the finite element method, and with a focus on understanding resonant responses. Following the majority of other similar modelling studies in this area (e.g. Holdsworth and Glynn, 1978, 1981; Vinogradov and Holdsworth, 1985; Fox and Squire, 1991; Sergienko, 2010; Bromirski and Stephen, 2012), we model the ice shelf as a thin elastic plate floating on an inviscid, irrotational fluid, and assume linear motions, on the basis of the long wavelengths/small amplitudes involved. Thus, the coupled ocean wave/ice shelf vibration problem is a sub-problem in the field of hydroelasticity, i.e. the study of the effects of fluids on elastic bodies, for which there exists a far larger corpus of mathematical modelling literature (e.g., see the review by Squire, 2008). The model is two-dimensional, with one horizontal dimension and one depth dimension, and the ice shelf is of finite length, clamped at the landward end and free at its seaward end. Ice shelf vibrations are excited by an incident

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wave from the open ocean, where the open ocean is modelled as a semi-infinite interval of free-surface water that abuts the ice shelf and sub-ice shelf water cavity. The shelf front partially reflects the incident wave, but the remaining proportion of the incident wave penetrates the shelf–cavity region, causing the shelf to vibrate. The vibrations radiate waves into the open ocean so that eventually all of the incident wave energy is scattered away from the shelf.

The finite element method is well suited to the analysis of hydroelastic systems, and Sergienko (2017) recently used commercial finite element code to compute vibrations of the Ross Ice Shelf. However, care needs to be taken in applying the finite element method when the ice shelf and sub-ice shelf–cavity are coupled to the open ocean, as this creates a semi-infinite scattering domain. We overcome this technical challenge by including a non-local linear operator boundary condition to represent the incident and scattered waves.

We use the finite element method in conjunction with an expansion of the ice shelf vibrations, using its in vacuo (dry) modes of vibration. The in-vacuo mode expansion has been used in other linear hydroelastic scattering problems (e.g. Bishop et al., 1986; Meylan and Squire, 1994; Newman, 1994; Meylan, 2002), as a generalization of the standard expansion for rigid bodies. It is the basis for the most sophisticated contemporary hydroelastic models, developed to analyse, for example, the motion of container ships (e.g. Hirdaris et al., 2003; Huang and Riggs, 2000; Senjanovic et al., 2008, 2009).

Vibrations of the floating ice shelf involve coupled elastic-fluid modes, which are difficult to calculate. Sergienko (2013) and Meylan et al. (2017) approximated them by applying a no-flux condition at the interface between the open ocean and the ice shelf–cavity, but it is not clear whether this gives a good approximation, and, further, we cannot say anything about how strongly they are excited or how quickly they decay in time without coupling to the surrounding ocean. Moreover, Sergienko (2013) and Meylan et al. (2017) considered shallow water. We use a method proposed in Wang and Meylan (2002), modified to allow for ice shelf submergence, to couple the cavity with the open ocean. This connection is critical to apply the finite element method correctly and would be difficult to implement in commercial finite element code, as the boundary condition is non-local and requires a linear operator.

Models of floating ice shelf vibrations typically assume shallow-water conditions (e.g. Sergienko, 2013; Meylan et al., 2017), on the basis of the long wavelengths. This assumption greatly simplifies the mathematical analysis of the problem and has been adopted in other areas of hydroelasticity (e.g. Zilman and Miloh, 2000; Meylan, 2002). The method developed here does not make this assumption, thus giving a process to validate the shallow water assumption. The problem of the motion of elastic plates on water of finite length has been the subject of extensive study and many different semi-analytic methods have been proposed (e.g. Meylan and Squire, 1994; Kohout et al., 2007; Bennetts et al., 2007; Gayen and Mandal, 2009; Behera and Sahoo, 2015; Renzi, 2016). Some of these methods could be adapted to model ice shelves, provided that some simplifying assumptions were made, the most significant which is that the depth under the ice shelf is constant. Such methods would complement the more numerical approach used in the current work. Other numerical approaches that relax the assumption of linearity could also be used, such as that of Guyenne and Părău (2017).

The most important practical calculation is the response of an ice shelf to a time-dependent forcing since the energy in the ocean travels in wave packets and it is the response in time which is measured. The frequency domain solution represents the equilibrium response to longtime forcing, and it is more appropriate to wave tank experiments (the basis for engineering tests). Moreover, care must be taken in interpreting the results from the frequency domain because very large responses in the frequency-domain are often difficult to excite in the time-domain.

The outline of this work is as follows. The equations of motion in the time domain and frequency domain are given in Section 2. In order to formulate a boundary value problem, we derive a boundary condition for the bounded domain using the wave incident from the semi-infinite domain in Section 4. We define the boundary condition in the ice shelf–cavity and the solution for the velocity potential in Section 5, using the added mass and damping matrix to solve the coupled problem. In Section 6, we introduce the finite element method and its discretization. We use the finite element method to calculate the potential solution associated with no motion and with each in vacuo mode. In Section 7 we show how to compute the time-dependent solution. Numerical results for frequency domain are presented in Section 8.1, and time-domain results in Section 8.2. Finally, Section 9 gives a brief summary.

2. Problem formulation

The goal of our work is to show how we can calculate vibrations of a floating ice shelf by using the finite element method. A critical technical difficulty that will be overcome is associated to the application of boundary conditions that couple the sub-shelf water cavity to the open ocean.

Fig. 1 shows a schematic of the problem. Positions in the water are described using the Cartesian coordinates $\mathbf{x} = (x, z)$, where x is the horizontal coordinate, and z is the vertical coordinate. The open ocean region is

$$\mathbf{x} \in \Omega^- = \{(x, z) : x < -L, -h_0 < z < 0\}, \tag{1}$$

where $z = 0$ is the equilibrium free surface of the open ocean, $z = -h_0$ is the seabed (assumed flat), and $x = -L$ is the location of the ice shelf front. The sub-shelf cavity region is

$$\mathbf{x} \in \Omega = \{(x, z) : -L < x < 0, -h(x) < z < -d\}, \tag{2}$$

where $z = -d$ is the underside of the ice shelf, $x = 0$ is the location of the landward end of the shelf, and $z = -h(x)$ is the location of the varying bed beneath the shelf. We assume that the depth of the water only varies under the ice shelf so that