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Semi-analytical unsteady aerodynamic model of a flexible thin airfoil

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HIGHLIGHTS

- A semi-analytical model for the unsteady aerodynamics of a flexible thin airfoil is proposed.
- The model generalizes mapping functions and the complex potential to arbitrary motions.
- Free wake modeled using discrete vortices is also included.
- The aerodynamic pressure load on the airfoil is evaluated analytically in space.
- Results for an airfoil with curvature of constant sign along the chord are presented.

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ABSTRACT

This paper presents a geometrically exact semi-analytical approach to model the unsteady aerodynamics of a flexible thin airfoil in incompressible potential flow. The velocity field is described by a complex potential, which is developed by mapping the airfoil boundary onto a circle. This methodology, well established for small disturbances, is here generalized to evaluate the aerodynamic load on an airfoil subject to prescribed arbitrary rigid-body motion and deformation. The model also includes the effects of free wake, modeled by a large number of discrete vortices. A general semi-analytical solution for the aerodynamic pressure distribution is obtained, specialized to the case of curvature of constant sign along the airfoil chord, and validated with reference linearized results. Next, the model validity for large curvatures is discussed by comparing velocity fields around increasingly deformed airfoils with numerical solutions from suitable panel methods. Finally, results for a cantilevered flexible airfoil subject to prescribed deformation in a steady axial flow are presented to point out different unsteady mechanisms influencing the aerodynamic pressure distribution and responsible for its changes in time. This insight is peculiar of the semi-analytical nature of the present formulation and cannot be easily obtained through numerical handling of the problem. The paper ends by outlining future developments of the present work toward the ultimate goal of studying coupled fluid–structure interaction problems in presence of very large deformations.

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Nomenclature

Latin symbols

$\boldsymbol{a}_k$	kth complex coefficient in $\tilde{\boldsymbol{\Phi}}^{(v)}$
b	half chord
$\mathcal{BC}_k$	kth branch cut of the map in the fluid domain
$\mathcal{BCP}_k$	preimage of $\mathcal{BC}_k$ in the auxiliary plane
$\boldsymbol{b}_k$	ratio $\boldsymbol{a}_k/\boldsymbol{w}_k$
ΔC_p	pressure jump coefficient
$\mathcal{C}$	unit circle in the auxiliary plane
$\mathcal{C}_r$	circle of radius $r > 1$ in the auxiliary plane
$\boldsymbol{c}_k$	kth Laurent coefficient in the map
$\boldsymbol{d}_k$	kth Laurent coefficient in $\tilde{\boldsymbol{\Phi}}^{(nc)}$
$\boldsymbol{h}$	undeformed airfoil centroid
$\boldsymbol{H}$	deformed airfoil centroid
$\boldsymbol{i}$	imaginary unit
k	non-dimensional Fourier variable (reduced frequency)
ℓ	undeformed airfoil length
m_k	modulus of $\boldsymbol{c}_k$
n	order of the map
N	number of shed vortices
$\boldsymbol{n}$	outer normal vector on the airfoil boundary
p	pressure
$\boldsymbol{p}_s$	complex coefficient in u_{b_n}
$\boldsymbol{P}_{3n}$	polynomial of degree $3n$ in $\boldsymbol{\omega}$
$\boldsymbol{q}_{r,j}$	complex coefficient in u_{b_n}
$\boldsymbol{Q}_{2n}$	polynomial of degree $2n$ in $\boldsymbol{\omega}$
r	radius of the circle $\mathcal{C}_r$ in the auxiliary plane
r_k	modulus of the kth singularity in $\tilde{\boldsymbol{\Phi}}^{(b)}$
s	arclength along the airfoil boundary
t	time
Δt	integration time step
Δt_g	vortex-shedding time step
T_m	period of oscillation of m_2
T_ϕ	period of oscillation of ϕ_2
u_{b_n}	component of $\boldsymbol{u}_b$ normal to the airfoil boundary
u_n	component of $\boldsymbol{u}$ normal to the airfoil boundary
u_∞	modulus of $\boldsymbol{u}_\infty$
$\boldsymbol{u}$	flow velocity in the airfoil plane
$\boldsymbol{u}_b$	velocity of the airfoil boundary
$\boldsymbol{u}_\infty$	freestream velocity
V_n	component of $\dot{\boldsymbol{H}}$ normal to the airfoil boundary
$\boldsymbol{w}_k$	kth additional singularity in $\tilde{\boldsymbol{\Phi}}^{(v)}$
x	horizontal coordinate in the airfoil plane
$\boldsymbol{x}$	position in the airfoil plane
$\boldsymbol{x}_b$	position on the airfoil boundary
$\boldsymbol{x}_v$	vortex position in the airfoil plane
y	vertical coordinate in the airfoil plane
$\boldsymbol{Y}_k$	kth critical point of the map in the fluid domain

Greek symbols

α	pitch angle
β	phase of $\boldsymbol{u}_\infty$
Γ_b	body circulation
Γ_{b_0}	initial body circulation
Γ_j	circulation of the jth vortex
$\boldsymbol{\gamma}_{jk}$	ratio $\Lambda_j/\overline{\boldsymbol{w}}_k$
δ	normalized distance of the nascent vortex from the airfoil trailing-edge