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A new analytical approach for modeling the added mass and hydrodynamic interaction of two cylinders subjected to large motions in a potential stagnant fluid

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ABSTRACT

A potential theory is presented for the problem of two moving circular cylinders, with possibly different radii, large motions, immersed in an perfect stagnant fluid. We show that the fluid force is the superposition of an added mass term, related to the time variations of the potential, and a quadratic term related to its spatial variations. We provide new simple and exact analytical expressions for the fluid added mass coefficients, in which the effect of the confinement is made explicit. The self-added mass (resp. cross-added mass) is shown to decrease (resp. increase) with the separation distance and increase (resp. decreases) with the radius ratio. We then consider the case in which one cylinder translates along the line joining the centers with a constant speed. We show that the two cylinders are repelled from each other, with a force that diverges to infinity at impact. We extend our approach to the case in which one cylinder is imposed a sinusoidal vibration. We show that the force on the stationary cylinder and the vibration displacement have opposite (resp. identical) axial (resp. transverse) directions. For large vibration amplitudes, this force is strongly altered by the nonlinear effects induced by the spatial variations of the potential. The force on the vibrating cylinder is in phase with the imposed displacement and is mainly driven by the added mass term. The results of this paper are of particular interest for engineers who need to understand the essential features associated with the vibration of a solid body in a still fluid.

1. Introduction

The interaction between a fluid and moving bodies is a fundamental problem which finds many applications, for example in turbomachinery (Furber and Ffowcswilliams, 1979), fish schooling (Nair and Kanso, 2007), heat exchangers tube banks (Chen, 1975, 1977), vibration of flexible risers (Le Cunff et al., 2002), biomechanics of plants (De Langre, 2008), or energy harvesting (Doare and Michelin, 2011; Singh et al., 2012; Michelin and Doare, 2013; Virot et al., 2016) from flapping flags (Eloy et al., 2008)… The reader should refer to the book of Païdoussis et al. (2014) for a complete bibliography of important works in this field. A classical approach to understand the fluid dynamics surrounding interacting bodies is to consider a potential flow model, in which viscous, rotational vortex and wake effects are disregarded. For large Reynolds numbers and

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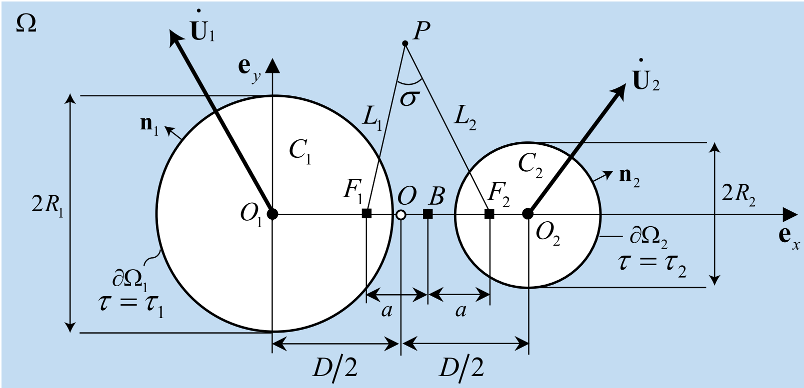

Fig. 1. Schematic diagram of the system: two moving circular cylinders $\mathcal{C}_i$ with radii R_i, centers O_i, velocities $\dot{\mathbf{U}}_i\,(T)$, are immersed in an inviscid fluid Ω. The motions of $\mathcal{C}_i$ generate an incompressible and irrotational flow. The instantaneous center-to-center distance is D. The bipolar coordinates $\sigma \in [0, 2\pi]$ and $\tau = \ln(L_1/L_2)$ are used to track the position of a fluid particle P. The points (B, F_1, F_2) are defined in such a way that points lying on the cylinder boundaries $\partial\Omega_i$ have bipolar coordinates $\tau = \tau_i$.

outside the boundary layers, the potential flow theory is expected to provide a good approximation of the solution, or some of its related characteristics as the added-mass, see Chen (1987). The very first discussions of potential flows around two circular cylinders probably originate from Hicks (1879), who studied the motion of a cylindrical pendulum inside another cylinder filled with fluid. To solve this problem, Hicks considered the potential due to flow singularities distributed over the fluid domain and the cylinder surfaces. The strengths and locations of the singularities were expressed as a set of iterative equations and the potential in an integral form, suitable for numerical computation (Lamb, 1945). Many investigators (Greenhill, 1882; Basset, 1888; Carpenter, 1958; Birkhoff, 1960; Gibert and Sagner, 1980; Landweber and Shahshahan, 1991) then tried to clarify and simplify the method of singularities by Hicks to derive a more tractable expression for the fluid potential. Still, this approximate method remains difficult to apply and as the gap between the cylinders becomes small, a high density of singularities is necessary for the solution to converge. More recently, analytical approaches based on complex analysis and conformal mapping methods (Wang, 2004; Burton et al., 2004; Tchieu et al., 2010) have overcome this problem and general theories to determine the flow through multiple bodies have been proposed (Scolan and Etienne, 2008; Crowdy, 2006, 2010). The two cylinders problem can be solved in this framework, but here we provide a more flexible method which yields new analytical and simplified expressions for the added mass coefficients. We also show that the large motions of the cylinders generate some strong spatial variations of the potential which produce nonlinear inertial effects of the fluid forces.

This paper is organized as follows. Section 2 presents the problem of two parallel circular cylinders immersed in an inviscid fluid and introduces the governing dimensionless numbers. In Section 3, we solve the potential flow problem, provide analytical expressions of the added mass terms and determine the fluid force on the cylinders. In Section 4 we test our predictions against published results and consider the case of a vibrating cylinder. Finally, some conclusions are drawn in Section 5.

2. Definition of the problem

Let $\mathcal{C}_i$, $(i = 1, 2)$, be two circular cylinders with radii R_i, immersed in an inviscid fluid of volume mass density ρ. We aim to determine the 2D flow generated by the arbitrary motions of $\mathcal{C}_i$, (Fig. 1). For convenience we introduce rescaled quantities to reduce the number of parameters of the problem. We use R_2 to normalize lengths, some characteristic speed V_0 to normalize velocities and $\rho {V_0}^2$ to normalize pressures. We define

$$r = \frac{R_1}{R_2}, d = \frac{D}{R_2}, \mathbf{u}_i = \frac{\mathbf{U}_i}{R_2}, t = \frac{TV_0}{R_2}, p = \frac{P}{\rho {V_0}^2}, \mathbf{f}_i = \frac{\mathbf{F}_i}{\rho {V_0}^2 R_2}, \tag{1}$$

as the radius ratio, the dimensionless center-to-center distance, cylinder displacement, time, fluid pressure and fluid force, respectively. We also introduce the dimensionless separation distance, $\varepsilon = d - (r + 1)$ such that $\varepsilon = 0$ corresponds to cylinders in contact.

3. Theoretical model

3.1. Fluid equations

It is assumed that the fluid is inviscid and the flow generated by the motion of the circular cylinders is incompressible and irrotational. We thus model the flow with a potential function φ which satisfies the Laplace equation

$$\Delta\varphi = 0 \quad \text{in } \Omega, \tag{2}$$