



Extension of Lighthill's slender-body theory to moderate aspect ratios

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ABSTRACT

Calculating the fluid forces acting on a moving body at high Reynolds number is crucial in many fluid–structure interaction problems, such as fish swimming or flutter instabilities. To estimate these forces, Lighthill developed the slender-body theory, which assumes a potential flow and an asymptotically small aspect ratio. Yet, it is still unclear whether Lighthill's theory is still valid for aspect ratios of order one. To address this question, we solve numerically with a panel method the full three-dimensional problem of a rectangular plate deforming periodically in a potential flow. These numerical simulations are used to calculate the pressure jump distribution across the plate for different aspect ratios. We find that numerical simulations and slender-body theory give similar results far from trailing edge. Close to the trailing edge however, there is a discrepancy, which is due to the use of a Kutta condition in the simulations (i.e. no pressure jump at the trailing edge), while, in the slender-body theory, the pressure jump is non zero. We propose a simple extension of Lighthill's slender-body theory that accounts for this discrepancy. The usefulness of this extension is then discussed and illustrated with a generic fluid–structure interaction problem and with the flag instability problem.

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1. Introduction

Slender-body theory has first been developed by Munk (1924) to study aerodynamic loads exerted on an airship hull. Munk (1924) considered an elongated rigid body moving in a fluid at rest. This theory is essentially based on potential flow theory and conservation of fluid momentum along the body axis. It implicitly assumes that the flow generated by each section does not affect the other sections of the body, which is guaranteed if the body is sufficiently slender and if the section does not vary abruptly.

In the context of fish swimming, Lighthill (1960) later extended slender-body theory to calculate the fluid forces exerted on a deforming body. In the present paper, we will refer to this extension as *Lighthill's slender-body theory*, or simply slender-body theory when there is no ambiguity. Its hypotheses are the same as in Munk (1924): the flow is assumed to be potential and incompressible, the body aspect ratio, defined as the ratio between crosswise and streamwise dimensions, is assumed to be asymptotically small. With these hypotheses, again, pressure forces exerted on the body can be calculated using an argument of momentum conservation. Remarkably, Lighthill (1960) showed that when a fish deforms periodically, the average total force only depends on the kinematics of the tail.

Slender-body theory is limited to displacements that are small compared to the streamwise body length. To circumvent this limitation, Lighthill (1971) extended slender-body theory, a theory known as *Lighthill's elongated-body theory*. Recently,

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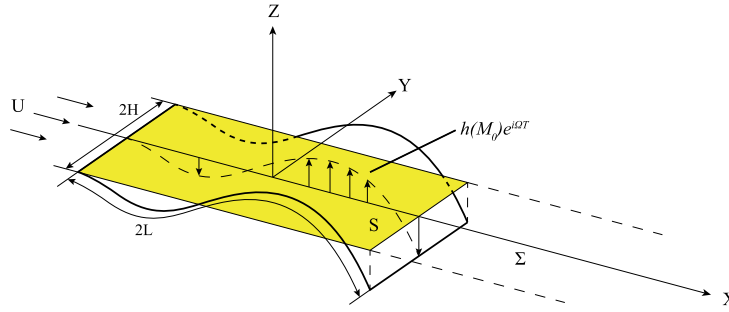


Fig. 1. Schematic representation of the lifting-surface problem. A flexible plate, with chord $2L$ and span $2H$, is deforming periodically in an incoming flow $\mathbf{U}$.

this elongated-body theory was itself extended by Candelier et al. (2011) and Candelier et al. (2013) to respectively account for arbitrary three-dimensional motions (including all components of rotation and translation) and a non-uniform background flow.

An important field of application of Lighthill's slender-body and elongated-body theories has been the hydrodynamics of fish swimming, as first advocated by Lighthill (1960, 1971). In this context, these theories have been used to better understand the kinematics and energetics of swimming, and in particular to assess whether fish employ smart drag reduction mechanisms or if, on the contrary, drag is enhanced by fish motions (Alexander, 1977; Webb, 1975; Videler, 1981; Ehrenstein and Eloy, 2013). To give an unbiased review of the field, we should also mention that some authors criticized Lighthill's theories, arguing that these potential approaches tend to overestimate forces (Hess and Videler, 1984; Anderson et al., 2001; Shirgaonkar et al., 2009). Lighthill's slender-body has also been applied to engineering problems, such as the flutter instability (Lemaitre et al., 2005), or energy harvesting (Singh et al., 2012; Michelin and Doaré, 2013). Note that Lighthill's slender-body theory should not be confused with the low-Reynolds-number slender-body theory (e.g., Lauga and Powers, 2009), even if – to add to the confusion – Lighthill also contributed to this field (Lighthill, 1976).

A significant limitation of Lighthill's slender-body theory is that it is only applicable to bodies of asymptotically small aspect ratios. In the context of the flutter instability, one of us addressed this limitation by developing a spectral approach (Eloy et al., 2007, 2008). In essence, the potential flow problem is first solved in the Fourier space for any aspect ratio; then, an inverse Fourier transform is applied to come back to real space. Unfortunately, this approach is limited to body with a rectangular planform and its implementation is not as simple as Lighthill's slender-body theory. The objective of the present study is to examine an alternative analytical method to treat bodies with moderate aspect ratios.

Another important drawback of Lighthill's slender-body theory is that it is not compatible with a Kutta condition at the trailing edge. Indeed, the Kutta condition (Crighton, 1985), even for unsteady flows, requires that the pressure jump across the lifting surface is zero at the trailing edge. In Lighthill's slender-body theory, not only the force at the trailing edge is finite, but its value is the only relevant quantity if one is interested in the average total force for a periodic motion.

This paper is organized as follows. In Section 2, we will recall how the lifting-surface problem is stated and we will describe an *asymptotic solution* of this problem valid in the limit of small aspect ratios. In Section 3, we will present a panel method in the frequency domain, aimed at solving the same problem numerically. An alternative analytical approach, also valid in the limit of small aspect ratios and called the *approximate solution*, will then be presented in Section 4. The mathematical derivation of this approximate solution may be less rigorous than the derivation of the asymptotic solution, but it is much simpler to implement and it provides a straightforward and useful correction to slender-body theory. The merits of these two different methods will be compared to slender-body theory and to the numerical panel method in Section 5 through a generic fluid–structure interaction problem and through the analysis of the flutter instability of a cantilever flexible plate. Finally, these results will be summarized and discussed in Section 6.

2. Asymptotic solution of the lifting-surface integral

Consider a uniform, incompressible, and irrotational flow of velocity $\mathbf{U}$ over a flexible plate deforming periodically with angular frequency Ω (Fig. 1). In a Cartesian coordinate system ($OXYZ$), the incoming fluid of density ρ is flowing along X , and the flexible surface is deforming around the reference plane $Z = 0$. The periodic deformation of the plate is described by the complex displacement $Z(M_0, T) = h(M_0)e^{i\Omega T}$, where M_0 is a point in the plane $Z = 0$. The displacement is assumed sufficiently small such that (1) vortex sheets attached to the flexible surface and shed in the wake can be represented by their projections onto the reference plane, respectively S and Σ (Fig. 1); (2) the flow induced by the motion of the flexible plate can be considered as a periodic perturbation of the incident flow. For the sake of simplicity, the complex factor $e^{i\Omega T}$ will be omitted for all unsteady quantities. The point M_0 will represent an arbitrary point in the plane S , the point M will be in S or Σ , and Q will be a point in the volume, outside S and Σ .

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