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A modal frequency-domain generalised force matrix for the unsteady Vortex Lattice method

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ABSTRACT

The unsteady Vortex Lattice method is becoming an increasingly popular aerodynamic modelling method for incompressible aeroelastic problems, such as flexible low-speed aircraft, wind turbines and flapping flight. It leads to discrete time aeroelastic state space equations, which must be solved in a time-marching framework. Eigenvalue or singular value decompositions of the discrete time equations can be used in order to perform stability analysis but such procedures must be accompanied by model order reduction because the size of the equations is large. This work proposes a modal frequency domain implementation of the Vortex Lattice method, resulting in a modal generalised force matrix. Model order reduction is implicit in the modal approach and stability analysis can be carried out using industry-standard flutter analysis techniques, such as the p–k method. The approach is validated by comparison to wind tunnel flutter data obtained from rectangular cantilever flat plate wings of different aspect ratios and sweep angles. It is found that the aeroelastic model predictions follow the experimental trends for both flutter speed and frequency but tend to be moderately conservative.

1. Introduction

The unsteady Vortex Lattice Method (Katz and Plotkin, 2001) has been used in aeroelastic simulations of incompressible flows by numerous researchers, with applications to rectangular plates (Tang et al., 1999a, 2003), Delta wings (Tang et al., 1999b; Tang and Dowell, 2001) or general wing geometries (Tang and Dowell, 2008; Varello et al., 2011; Murua et al., 2012; Haghighat et al., 2012). Of particular interest are applications to very flexible low-speed aircraft (Palacios et al., 2010), such as sensorcraft and High Altitude Long Endurance planes, wind turbines (Pesmajoglou and Graham, 2000; Prasad et al., 2017) or even flapping wings (Stanford and Beran, 2010; Dimitriadis et al., 2015). Such aerodynamic models are always formulated in discrete time, which is not suitable for stability analysis and does not reflect the standard industrial practice of frequency domain aerodynamic models, as they are obtained from the Doublet Lattice Method (Albano and Rodden, 1969). Tang et al. (1999a, b) and others used an eigenvalue decomposition of the discrete time problem in order to carry out stability analysis but the procedure requires model order reduction, as the size of the matrix coefficients of the discrete time equations is large. Rule et al. (2001) converted the discrete time equations to continuous time via a series solution but they also resorted to model order reduction. Tang et al. (2001) applied several different decomposition approaches to the discrete time unsteady aerodynamic problem, such as Singular Value Decomposition and Proper Orthogonal Decomposition, in order to obtain reduced order models and assess stability.

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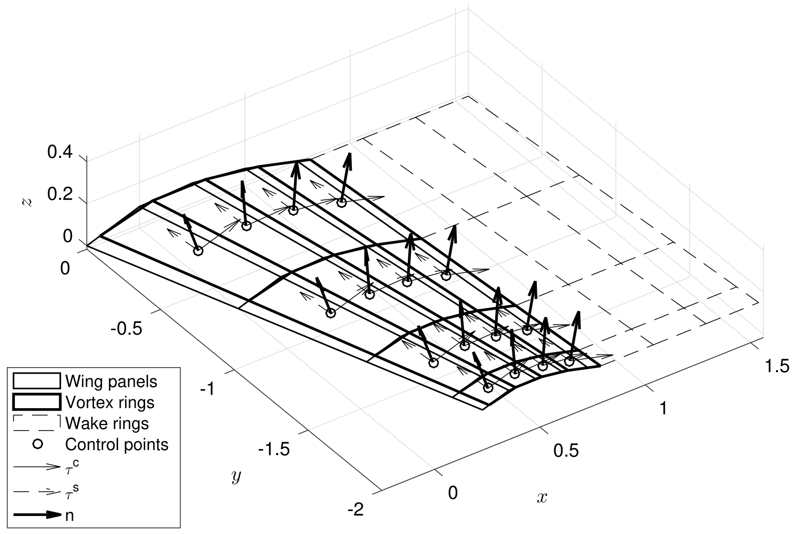

Fig. 1. Wing discretisation for the Vortex Lattice method.

In this work, a modal frequency domain version of the Vortex Lattice Method is developed, leading to a modal Generalised Force matrix. The discussion here is a generalisation of the methodology described in Dimitriadis (2017). The resulting aeroelastic model can be solved using all the usual flutter analysis approaches, such as the k and p-k methods or Rational Function Approximations. Hence, the Vortex Lattice Method can be re-cast in the standard language of practical aeroelasticity.

2. Mathematical development

Consider a wing structure whose out-of-plane displacement, $z(x, y, t)$, can be described by a modal expansion of the form

$$z(x, y, t) = \sum_{i=1}^{n_m} w_i(x, y) r_i(t) \tag{1}$$

where n_m is the total number of modes, $w_i(x, y)$ is the ith mode shape and $r_i(t)$ the ith generalised coordinate. Equations of motion for this wing can be set up using Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{r}_i}\right) - \frac{\partial E}{\partial r_i} = Q_i \tag{2}$$

for $i = 1, \ldots, n_m$, where $E = T - V$ is the total energy in the structure, T is the kinetic energy and V is the potential energy (see for example Meirovitch (1980)). The left-hand side of Lagrange's equation gives the internal forces in the structure in terms of the generalised coordinates, while the right-hand side represents the generalised aerodynamic forces, given by

$$Q_i(t) = \int_S \Delta p(x, y, t) \frac{\partial z}{\partial r_i} dS \tag{3}$$

where $\Delta p(x, y, t)$ is the pressure difference acting on point x, y at time t and S is the area of the wing.

Fig. 1 demonstrates the discretisation scheme for the wing and wake, which follows the guidelines given by Katz and Plotkin (2001). Geometric panels are defined on the mean (camber) surface of the wing and vortex rings are placed on them, such that the leading edge of each vortex ring coincides with the quarter-chord line of each panel. The impermeability boundary condition is imposed on control points placed on the three-quarter-chord of each panel. Vortex rings are also placed in the wake behind the wing, the leading edge of the first wake ring coinciding with the trailing edge of the last bound vortex ring. The wake is necessary in order to impose the unsteady Kutta condition (Katz and Plotkin, 2001). Therefore, the wing is discretised into m chordwise and n spanwise panels and its wake is described by m_w chordwise and n spanwise vortex rings. The wing is immersed in a free stream with velocity $\mathbf{U}_\infty = [U \ V \ W]$, i.e. with airspeed $Q_\infty = |\mathbf{U}_\infty|$ and direction $\hat{\mathbf{u}} = [U \ V \ W]/Q_\infty$. The wake is frozen and propagates in the streamwise direction with the free stream airspeed. The chordwise spacing of the wake vortex rings is chosen as $\bar{c}/m$, where $\bar{c}$ is the mean chord.

The geometries of the wing and wake remain unchanged (frozen) throughout the time history. Structural motion is represented only aerodynamically, by introducing a downwash term caused by structural motion in the z direction (in-plane