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# Experiments on the synchronous sloshing in suspended containers described by shallow-water theory

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ABSTRACT

The synchronous interaction between a sloshing fluid and its container may take place when the motion of the container is not prescribed *a priori*. When a partially-filled container is suspended as a bifilar pendulum, the fluid contributes to the restoring force by its weight through the wire suspensions and may either augment or diminish the restoring force through hydrodynamic interaction with the container. Here we report on linearized shallow-water theory and experiments for multi-chamber box containers, a cylinder and an upright 90° wedge. The nature of system response at low fluid mass is analyzed in detail using linear theory and shallow-water simulations. These results show that the system frequency smoothly transitions through successive higher frequency eigenmodes so that, in the limit where the fluid mass tends to zero, the system frequency is that of the dry container.



## 1. Introduction

The dynamics of containers partially-filled with liquid dates back to the classical work of nineteenth-century scientists. In the mid-twentieth century, interest in such problems was revived owing to a number of technical applications ranging from seismic oscillations of fuel reservoirs to the dynamics of aircraft and rockets taking into account fuel which they carry. These problems are distinct from the problem of fluid oscillations brought about by a prescribed motion of the container: in the unconstrained or partially constrained problems of interest here the motion of the container depends crucially on the dynamics of the fluid within. A theory for the oscillations of a beam with a liquid-containing cavity was reported by Moiseev (1964) and other problems of this type related to space vehicle technology may be found in a NASA publication edited by Abramson (1966).

An article by Cooker (1994) considers the interesting problem of fluid-container interaction when the container is suspended as a bifilar pendulum with equal suspension lengths $l$. For small nondimensional displacements $X_0/l$ of the pendulum about equilibrium, the motion is close to being horizontal only, with the vertical deflection of the container being of order $(X_0/l)^2$. The natural oscillation frequency for a dry container is the pendulum frequency $\omega_0 = \sqrt{g/l}$, where $g$ is the gravitational constant. Cooker's experiments performed with a container partially filled with water and released away from equilibrium showed that large amplitude waves are generated at the sidewall during the first swing. These waves dissipate with each subsequent swing and eventually the system settles down to periodic motion at frequency $\omega_1 < \omega_0$, with the fluid

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moving synchronously in-phase with the pendulum. Other initial conditions gave synchronous fluid motion exactly out-of-phase with the pendulum in which case the system oscillated at frequency $\hat{\omega}_1$, such that $\omega_1 < \hat{\omega}_1$.

Cooker (1994) formulated two problems, each helping to understand the above observations. In the first, two-dimensional linearized shallow-water theory was employed to analyze how sloshing motions in a tank of uniform rectangular cross-section interact with the gravitational restoring force to give a discrete spectrum of wave modes synchronized with the motion of the container. A transcendental equation obtained for the wavenumber of fluid sloshing was solved numerically using Newton iteration. Results for the fundamental frequency of in-phase motion were shown to be in accord with the seiche frequency observed in experiments. In the second problem, linearized potential theory with the hydrostatic pressure assumption was used to find planar geometries admitting synchronous solutions for suspended containers. The container shapes uncovered were a family of hyperbolae including, as a limiting container form, a wedge with 90° vertex angle. Both in-phase frequencies $\omega_1$ and anti-phase frequencies $\hat{\omega}_1$ of a planar free surface oscillating about its central nodal line were reported, and it was shown that $\omega_1 < \hat{\omega}_1$.

It may be noted that the limit $l \to \infty$ for the pendulum supported system corresponds to the *free motion* of a fluid-filled container wherein the sole driving force is the fluid sloshing against the container walls. Potential flow theory and corresponding experiments carried out for this type of motion in boxes, cylinders, wedges, cones, and cylindrical annuli have been reported by Herczynski and Weidman (2012).

In his single-chamber calculations, Cooker (1994) reported oscillation frequencies that depend on two parameters: $G = (1 + R)D^2/lH$ and $R = m_0/m$, in which $m_0$ is the mass of the dry container, $m$ the fluid mass, $D$ the half-length of the container and $H$ the mean fluid depth. While the dimensionless parameters $G$ and $R$ conveniently exhibit the graphical solution of the eigenvalue equation, they are not independent since $G$ depends on $R$ both explicitly and implicitly though the fluid depth $H$. In this paper we determine new independent parameters using dimensional analysis.

The shallow-water theory of Cooker (1994) was extended to non-shallow depth fluids by Yu (2010) who found that in this scenario the rectangular and cylindrical container eigenmodes consist of the shallow-water eigenmode plus a sum of vertical eigenmodes (Linton and McIver, 2001). Results were presented showing the dramatic effect of finite, but non-shallow, fluid depths. Alemi Ardakani et al. (2012) found that the rectangular container can exhibit a 'resonance' behavior, where anti-symmetric fluid eigenmodes, which couple to the vessel motion, can have the same oscillation frequency as the symmetric fluid eigenmodes, which exhibit zero force on the container. These ideas were also examined for multi-chamber rectangular containers in Turner et al. (2013).

In the present investigation it is assumed that each container examined is suspended as a bifilar pendulum and we consider fluid depths for which the motion may be adequately modeled using shallow-water theory. An exception is the motion of a 90° wedge for which shallow-water theory does not apply. In this case we derive the oscillation frequency of the wedge suspended as a bifilar pendulum using potential theory. Also we focus on in-phase sloshing motions which are readily obtained in laboratory experiments by simply pulling back the container from equilibrium and releasing it gently with the fluid initially at rest.

In Section 2 we extend the constant depth shallow-water analysis of Cooker (1994) for sloshing in a channel of rectangular section to a multi-chamber container suspended as a bifilar pendulum. In Section 3 the sideways sloshing of fluid in a vertically suspended cylindrical container is considered in the context of linearized shallow-water theory where solutions for synchronous oscillation in terms of Bessel functions are obtained. Potential theory for the 90° wedge is presented in Section 4. Experimental measurements for each of the above containers are presented in Section 5. In Section 6 we present experimental results and theory for the case of small fluid-to-container mass ratios, $m/m_0$, showing a transition through higher frequency eigenmodes as this ratio is reduced. A discussion of results and concluding remarks are given in Section 7.

## 2. Multicompartment containers

An interesting question of wavenumber selection arises when one considers the motion of a single container divided into different compartments partially filled with fluid and suspended as a bifilar pendulum. The situation is sketched in Fig. 1. The problem is solved by a straightforward extension of the shallow-water analysis given by Cooker (1994) for a single rectangular channel, but now $N$ compartments of arbitrary length, width, and quiescent fluid depth are considered. The resulting characteristic equation for the system frequencies was first derived by Turner et al. (2013), but here we recreate this derivation in our notation We denote the compartment widths $W_n$, compartment lengths $L_n = 2D_n$, quiescent fluid depths $H_n$, fluid densities $\rho_n$, compartment fluid masses $m_n = \rho_n W_n L_n H_n$, and the fixed container mass is $m_0$.

The fluid motion in each compartment is assumed to be governed by the linear, shallow-water equations

$$\frac{\partial \eta_n}{\partial t} = -H\frac{\partial u_n}{\partial x}, \quad \frac{\partial u_n}{\partial t} = -g\frac{\partial \eta_n}{\partial x} \tag{2.1}$$

where $u_n$ is the absolute horizontal velocity of the fluid, independent of the depth in the shallow-water approximation, $\eta_n$ is the free surface displacement, $x$ is a horizontal space coordinate in the moving frame, measured from the center of the container and $t$ is time.

Solutions are sought for which the container moves in simple harmonic motion according to