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Liquid sloshing in a horizontally forced vessel with bottom topography

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ABSTRACT

This paper presents a numerical study of the free-surface evolution for inviscid, incompressible, irrotational, horizontally forced sloshing in a two-dimensional rectangular vessel with an inhomogeneous bottom topography. The numerical scheme uses a time-dependent conformal mapping to map the physical fluid domain to a rectangle in the computational domain with a time-dependent aspect ratio $Q(t)$, known as the conformal modulus. The advantage of this approach over conventional potential flow solvers is the solution automatically satisfies Laplace's equation for all time, hence only the integration of the two free-surface boundary conditions is required. This makes the scheme computationally fast, and as grid points are required only along the free-surface, high resolution simulations can be performed which allows for simulations for mean fluid depths close to the shallow water water regime. The scheme is robust and can simulate both resonate and non-resonate cases, where in the former, the large amplitude waves are well predicted.

Results of nonlinear simulations are presented in the case of non-breaking waves for both an asymmetrical 'step' and a symmetric 'hump' bottom topography. The natural free-sloshing mode frequencies are compared with the small topography asymptotic results of Faltinsen and Timokha (2009) (*Sloshing*, Cambridge University Press (Cambridge)), and are found to be lower than this asymptotic prediction for moderate and large topography magnitudes. For forced periodic oscillations it is shown that the hump profile is the most effective topography for minimizing the nonlinear response of the fluid, and hence this topography would reduce the stresses on the vessel walls generated by the fluid. Results also show that varying the width of the step or hump has a less significant effect than varying its magnitude.

1. Introduction

Being able to accurately predict the free-surface motion of a fluid in a vessel is of practical importance in physical applications. The sloshing effects of a fluid in an externally forced vessel may cause detrimental consequences in many engineering applications. For example, the sloshing of liquid fuel in the fuel tanks of spacecraft or rockets can affect their trajectory or, if the sloshing frequency is close to the natural sloshing frequency of the fuel tank itself, then the high dynamic pressures caused by the resonating fluid could damage the walls of the tank. For more information on aerospace applications see the works of Abramson (1966) and Gerrits (2001).

In general, a three-dimensional vessel, such as a ship floating on the ocean, has 6 degrees of freedom. It has 3 linear translations heave, sway and surge and 3 rotational motions, pitch, roll and yaw. Understanding the response of the vessel, and the fluid it contains, to each of these 6 different degrees of freedom is vital to fully understand the stability properties of

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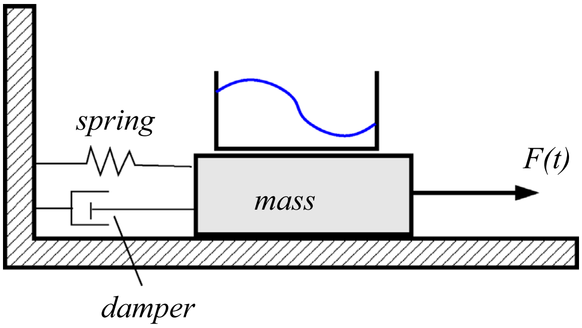

Fig. 1. A schematic illustration of a Tuned Liquid Damper (TLD).

the coupled system (Faltinsen and Timokha, 2009). In the present work we simplify this general situation to consider only 1 linear translation degree of freedom and focus on understanding the free-surface evolution caused by horizontal vessel motion only, such as in Tuned Liquid Dampers (TLDs).

Tuned Liquid Dampers are vessels which contain a fluid which are designed to suppress wind and earthquake oscillations in tall buildings (Kareem et al., 1999). A schematic of a TLD is given in Fig. 1. The designers of TLDs are responsible for understanding the complicated dynamic coupled motion of the fluid-vessel interaction in order to determine the optimal amount of fluid in the TLD to damp the most severe oscillations. Such an investigation would be costly via experiments alone, hence having an effective numerical scheme which can simulate various forcing frequencies and vessel topologies is beneficial.

Studying the sloshing motion in a stationary or forced vessel either experimentally, theoretically or numerically is very complicated. The works of Moiseyev and Rumyantsev (1968), Ibrahim (2005) and Faltinsen and Timokha (2009), and the references herein, highlight many of the problems observed in this area. The main theoretical and numerical difficulty is accurately calculating the position of the free-surface, which is unknown of the problem. Previous studies of this problem have tended to use one of two approaches for following the evolution of the free-surface. The first approach uses Lagrangian particle tracking of the numerical nodes on the free-surface with regridding, but the disadvantage of this approach is that the surface velocities are difficult to accurately calculate and so the surface has to be smoothed. The second approach uses mappings to map the physical domain to a rectangular computational domain with the free-surface now aligned with one edge of the rectangle. This approach requires no smoothing but it cannot easily predict flow features such as wetting and drying of the vessel bottom. However, for most physical applications this is not a major restriction of the method. This mapping approach was successfully implemented by Frandsen (2004) who used a σ-transformation to map the liquid domain onto a fixed rectangular computational domain (Phillips, 1957) for two-dimensional, inviscid, incompressible, irrotational sloshing in a rectangular vessel with a flat bottom. In the computational domain a transformed version of Laplace's equation was solved on a rectangular grid for the velocity potential ϕ, with the appropriate boundary conditions on the vessel walls and the free-surface. Frandsen (2004) demonstrated that this approach was successful for a flat bottomed rectangular vessel by verifying free-surface results against weakly nonlinear asymptotic results. However, this σ-transformation approach is limited in two ways: firstly, by solving the transformed version of Laplace's equation directly in the interior of the domain, a two-dimensional numerical grid is required, thus restricting the computational resolution of the method, and secondly, the given σ-transformation does not extend easily to vessels with inhomogeneous bottom topographies. The numerical approach used in this paper overcomes both these shortfalls and thus is significant to research in this area.

The numerical scheme used in this paper uses a time-dependent conformal mapping to map the physical domain to a rectangular computational domain with time-dependent aspect ratio, $Q(t)$, known as the conformal modulus. As the mapping is conformal the coordinates in the physical domain $x(\mu,\nu,t)+\mathrm{i}y(\mu,\nu,t)$ and the complex potential $\phi(\mu,\nu,t)+\mathrm{i}\psi(\mu,\nu,t)$ are time-dependent holomorphic functions of the computational domain coordinates (μ,ν), so they satisfy the Cauchy–Riemann equations

$$x_\nu = -y_\mu, \quad x_\mu = y_\nu, \quad \phi_\nu = -\psi_\mu, \quad \phi_\mu = \psi_\nu, \tag{1.1}$$

and hence they all satisfy Laplace's equation

$$x_{\mu\mu}+x_{\nu\nu}=0, \quad y_{\mu\mu}+y_{\nu\nu}=0, \quad \phi_{\mu\mu}+\phi_{\nu\nu}=0, \quad \psi_{\mu\mu}+\psi_{\nu\nu}=0, \tag{1.2}$$

in the computational domain. Here ψ is the corresponding streamfunction to the velocity potential ϕ. Because of (1.1) and (1.2) we can construct a numerical scheme such that we calculate only the evolution of the two harmonic functions y and ϕ on the free-surface, and use integral transforms to relate these functions to the conjugate harmonic functions x and ψ along the free-surface (Turner and Bridges, 2015; Dyachenko et al., 1996, 1999; Choi and Camassa, 1999). When the fluid depth in the vessel is infinite, the integral transforms are just the Hilbert transform (Dyachenko et al., 1996; Papamichael and Stylianopoulos, 2010), but for finite depth fluids the transforms are given by the Hilbert–Garrick transform and depend upon the conformal modulus $Q(t)$ making the transforms time dependent (Turner and Bridges, 2015). The importance of