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Capture into slow-invariant-manifold in the fluid–structure dynamics of a sprung cylinder with a nonlinear rotator

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ABSTRACT

We investigate the dynamics of a two-dimensional circular cylinder mounted on a linear spring, restricted to move in the cross-flow direction and undergoing vortex-induced vibration, incorporating a strongly nonlinear (i.e., non-linearizable) internal element consisting of a mass that is free to rotate about the cylinder axis and whose angular motion is restrained by a linear viscous damper. The conjunction of the essentially nonlinear inertial coupling with the dissipative element makes the internal attachment behave as a nonlinear energy sink that is able to extract and dissipate energy from the motion of the cylinder and (indirectly) the surrounding fluid. At the intermediate Reynolds number $Re = 100$, we find that the cylinder with rotator undergoes repetitive cycles of slowly decaying oscillations interrupted by chaotic bursts; during the slowly decaying portion of each cycle, the dynamics of the cylinder is regular and can lead to significant vortex street elongation with partial stabilization of the wake. We construct a reduced-order model of the fluid–structure interaction dynamics based on the data obtained by direct numerical simulation, and employ analytical techniques such as complexification/averaging and the multiple-scales method to show that the strongly modulated response is the manifestation of a resonance capture into a slow invariant manifold (SIM) that leads to targeted energy transfer from the cylinder to the rotator. Capture into the SIM corresponds to transient cylinder stabilization, whereas escape from the SIM leads to chaotic bursts. Hence, the action of the nonlinear rotator on the resonance dynamics of the fluid–structure interaction is clarified.

1. Introduction

Flow separation aft of an elastically supported bluff body results in shedding of vortices, thus generating an oscillating lift force that excites the body in such a way that it undergoes large amplitude vortex-induced vibrations (VIV) (Bearman, 1984, 2011; Williamson and Govardhan, 2004). Structures that are subjected to VIV can be found in a broad range of engineering

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applications; e.g., tall chimneys, power transmission lines and off-shore structures. In most practical cases, suppression of VIV achieved using passive or active approaches is sought in order to reduce or even completely eliminate excessive vibrations and dynamic instabilities (Owen et al., 2001; Bernitsas and Raghavan, 2008; Assi et al., 2010). In other cases, for instance in the context of energy harvesting and power pick-off, the effects of VIV can be beneficial, since it can be incorporated into designs to achieve efficient harvesting in flow–structure interaction regimes. In such applications, focus is directed toward enhancing the flow-induced vibrations in order to achieve large-amplitude motions that, in turn, may have implications for efficient energy harvesting (Bernitsas et al., 2008; Barrero-Gil et al., 2010, 2012; Grouthier et al., 2013, 2014).

A prototypical VIV model consists of the flow past a circular cylinder mounted on a grounded linear spring, and constrained to move in the cross-flow direction. Recent studies have investigated the dynamics of a system in which the dimensionality of the standard VIV configuration is augmented by coupling the cylinder with an essentially nonlinear dissipative attachment (or nonlinear energy sink, NES), which is free to translate and/or rotate inside the cylinder (with no gravitational effects) (Tumkur et al., 2013, 2013; Tumkur, 2014). The NES, because it features a dissipative element and strong inertial nonlinearity, has the capability to absorb vibrational energy from the primary structure to which it is attached, over broad frequency and energy ranges and for various types of narrowband or broadband excitations. For instance, it has been applied for passively mitigating aeroelastic instabilities in an aircraft wing (Gendelman et al., 2010). Moreover, the one-way, almost irreversible, nonlinear targeted energy transfer (TET) from a primary structure to the NES is related to cascades of transient resonance captures on slowly invariant resonance manifolds (Arnold et al., 1988; Vakakis et al., 2008; Sigalov et al., 2012, 2012).

The addition of a rotational NES to the transverse VIV model has been shown to result in drastic passive suppression of vibrations of the cylinder at intermediate Reynolds number ($60 \leq Re \leq 120$) (Tumkur, 2014). Although there is no *direct* interaction between the NES and the infinite-dimensional flow, the rotational attachment has an *indirect* effect on the flow, i.e., through the dynamics of the cylinder to which it is inertially coupled (Tumkur et al., 2016). One notable VIV suppression mechanism of particular interest that has been identified consists of repetitive cycles of slowly decaying oscillations of the cylinder followed by intermediate chaotic bursts (Tumkur et al., 2016). This interesting nonlinear dynamical phenomenon has been attributed to the presence of the rotating internal NES, since no similar results have been found for the case of the translational NES (Tumkur, 2014; Tumkur et al., 2013). Moreover, during the slowly decaying part of the cycle, the rotational NES performs phase-locked steady rotations, and this is accompanied by a noticeable reduction of the lift and drag coefficients of the cylinder, as well as profound effects on the wake; in particular, vortex street elongation occurs, resulting in partial stabilization of the wake behind the cylinder.

In this work, we aim to show that the repetitive cycles of slowly decaying cylinder motions interrupted by chaotic bursts result from successive resonance captures on and escapes from a slow invariant manifold of the dynamics, during which the cylinder, the pendulum and the flow—meaning, the lift force—are locked into a regime of 1:1:1 resonance, thus leading to targeted energy transfer from the surrounding fluid and the oscillating cylinder to the NES. We first perform an extensive series of direct numerical simulations (DNS) to identify sets of NES parameters for which such cycles are realized at $Re = 100$, and observe that this behavior is found over a relatively broad range of NES parameters. Next, we analytically study the dynamics of the system in the regime of the aforementioned resonance where the succession of slowly decaying cycles occurs. In our asymptotic analysis, we focus on cases where the ratio of the NES mass to that of the sprung cylinder is either small or finite. For small values of this mass ratio, we show that, after proper complexification/averaging of the equations of motion, the resonant fluid–structure interaction dynamics is amenable to multiple-scales analysis. For larger values of the mass ratio, however, we rely on a slow-fast partition of the dynamics to study the slowly decaying cycle. In both cases, we find good agreement between direct numerical simulations of the full-order system and the approximate analysis, thus demonstrating that the slowly decaying cycles interrupted by chaotic bursts are caused by successive resonance captures on and escapes from an underlying slow invariant manifold of the 1:1:1 resonant dynamics. The analytical results then pave the way for predictive design of the internal nonlinear element (in this case the rotator) for optimal flow stabilization.

2. Computational model and motivation

2.1. Physical model and governing equations

We consider a two-dimensional circular cylinder of mass M_{cyl} and diameter D, immersed in an incompressible fluid of density ρ_{f} and kinematic viscosity ν. The cylinder is mounted on a linear spring of stiffness K_{cyl}, and allowed to move only in the direction transverse to the free stream. A pendulum of mass M_{NES} is attached to the cylinder and freely rotates at constant radius r_0 about the cylinder's generatrix. Inertial coupling allows energy to be transferred from the cylinder (and indirectly from the surrounding fluid) to the eccentric rotator, part of which is dissipated by a linear rotational viscous damper about the axis of rotation of the NES.

The flow is governed by the Navier–Stokes equations, which we write in dimensionless form (asterisk denoting dimensionless quantities) as