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Free vibrations of moderately thick truncated conical shells filled with quiescent fluid

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ABSTRACT

A novel reduced order formulation is proposed for the vibration analysis of conical shells containing stationary fluid. Hamiltonian approach is followed to obtain the governing equations of motion for the structure. Utilizing the Navier–Stokes equations and simplifying for irrotational, compressible and inviscid assumptions, the final fluid equation is obtained. A general solution based on the Galerkin method is proposed for the conical shell in vacuum. Several boundary conditions are investigated to show the capability of the proposed solution. A novel reduced order formulation based on the finite element method is developed for solution of the fluid equation. Static condensation technique is also utilized to minimize the required number of degrees of freedom and speed up the solution. The main advantage of the current solution method is the use of minimal number of degrees of freedom yet giving highly accurate results. Effects of added mass, semi-vertex angle, boundary conditions and different fluid containments on the natural frequencies of the coupled-field problem are studied and some useful conclusions are drawn.

1. Introduction

Shell structures are widely used in various engineering applications with several geometries and configurations. Conical shells are a broadly accepted geometry which have a variety of different applications in civil, petroleum, marine and aerospace industries. Shell structures are commonly in contact with some sort of internal or external fluids. Taking the fluid hydro/aerodynamic loadings into account, dynamic characteristics of the structure can alter significantly. Hence, a great deal of studies are devoted to fluid structure interaction of shell-type structures in the past two to three decades. Although conical shells are one of the most general shell-type geometries, their interaction with internal fluids have not received sufficient attention. Dynamic behavior of conical shells in vacuo has been the subject of many studies since the sixties (Kempner and Axisymmetric, 1964; Irie et al., 1982; Tong, 1993; Tornabene, 2009; Shu, 1996; Liew et al., 2005; Civalek, 2013) while the first fluid loaded studies of conical shells date back to the nineties by Ulitin (1991). He studied the free vibrations of isotropic thin conical shells fully-filled with fluid and proposed an analytical formula to compute the fundamental frequency as a function of geometric parameters of the shell. In a later study by Jeong et. al. (1999) a theoretical formulation is suggested for the free vibrations of a thin conical shell filled with an ideal fluid at both simply supported and clamped

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boundary conditions via a finite element approach. Dynamic behavior of anisotropic conical shells with internal fluid is studied by Lakis et al. (1992). A novel finite element approach is introduced where conical finite elements are used to discretize the domain. The velocity potential formulation and Bernoulli equation are employed to obtain the hydrodynamic loading. Jhung et al. (2006) also studied the fluid–structure interaction of bridged conical shells filled with ideal fluid. A velocity potential formulation is used to model the fluid inside the shell and a finite element solution is provided for the coupled system of equations. Caresta and Kessissoglou (2008) have developed an analytical model to study the modal vibrations of fluid loaded conical shells in the low frequency range. Solution to the shell equation is presented in a power series format. The fluid loading is also obtained by dividing the shell into a finite number of narrow strips which are considered to be locally cylindrical with a specific mean radius. Such a model is identified to be feasible in the low frequency range.

Some of the recent studies are devoted to the shells conveying internal fluid flows. In the first study, Kumar and Ganesan (2008) proposed a semi-analytical finite element formulation for elastic conical shells conveying water. They used Bernoulli equation to determine the pressure acting on the walls. In addition to free vibration characteristics of the coupled-field problem, they obtained stability margins and critical fluid velocities. In a later study by Kerboua et al. (2010) a hybrid finite element method is proposed to study vibrations of conical shells with internal flowing fluid. In their proposed methodology, the displacement functions are derived from exact solutions of Sanders' equilibrium equations. The fluid flow is modeled by velocity potential equation and the pressure acting on the wall is estimated by Bernoulli equation. The hybrid finite element solution is eventually shown to be consistent with the semi-analytical solution provided by Kumar and Ganesan (2008) except for the critical velocities. The third major contribution in vibrations of conical shells with flowing internal fluid is the recent study by Bochkarev and Matveenko (2011). They used finite element method to solve both structural and fluid equations. Stability analysis of both cylindrical and conical shells are as well provided where major differences compared with the previous results of Kumar and Ganesan (2008) and Kerboua et al. (2010) are reported. Effects of different boundary conditions of the velocity potential equation are investigated and new insights are provided.

In the present study, a conical shell with internal quiescent fluid is considered. Governing equations of motion are obtained in their most general form by means of variational principle. Effects of rotary inertia and shear deformation are taken into account and an extra modification (known as Sanders modification) is as well applied to the governing equations in order to remove rigid body rotations in the solution. An efficient and incredibly fast solution method based on the Galerkin approach is employed to solve the structural equations of motion. The idea utilized in the solution of the structural equations of motion was first introduced by Su et al. (2014). The fluid is considered to be irrotational, inviscid and compressible, hence the governing equation is obtained via simplification of the Navier–Stokes equation to get the so-called wave/acoustic equation. A novel reduced order model (ROM) formulation is introduced along with static condensation technique to obtain the most efficient solution in terms of accuracy and computational effort. The final coupled field solution is significantly faster than other conventional solution algorithms, i.e. finite element method, since the number of required degrees of freedom is minimized. Verification studies are conducted for both conical shells in vacuo and fluid loaded shells and some parametric studies concerning the compressibility, boundary condition and semi-vertex angle effects on the dynamic behavior are performed.

2. Governing equations

2.1. Structural equations of motion

The most general and accurate models for the vibrations of any continuous system can be obtained by using the strain-displacement relations of 3D elasticity (Reddy, 2007; Sokolnikoff, 1956). Although such models can be highly accurate, the governing equations obtained are very complex to solve and even if solutions are obtained for specific problems, the computational cost is considerably high. The problem becomes even worse for shell structures where the strain–displacement relations are written in the curvilinear coordinates. Different shell theories are proposed in the literature based on the 3D elasticity relations to minimize aforementioned problems as well as maintaining a reasonable accuracy in solutions. The basic idea behind any shell theory is to reduce the 3D problem to an equivalent 2D representation by approximating the displacements across the thickness and then solving for the displacements on the mid-surface. A comprehensive monograph on the shell theories, their distinctions, advantages and disadvantages is provided by Leissa (1973).

All shell theories can be classified into one of the following categories, Classical Shell Theories (CST), First order Shear Deformable Theories (FSDT) and Higher order Shear Deformable Theories (HSDT). Classical shell theories are fundamentally developed for thin shells according to the Kirchhoff–Love kinematic assumption. This assumption expresses that the straight lines normal to the undeformed mid-surface remain straight and normal to the mid-surface after deformation. Therefore, the effects of shear deformation and rotary inertia are neglected in theories utilizing this assumption. Depending on different assumptions made during the derivation of strain-displacement relations or even in the determination of force and moment resultants, different shell theories are developed within the framework of CST such as Donnell's, Love's, Reissner's, Novozhilov's, Vlasov's, Sanders's and Flugge's thin shell theories. As mentioned, CSTs are only reliable and accurate for thin shells in their lower frequency range and in order to increase accuracy and reliability, a second category of shell theories are proposed to take the shear deformations into account. Actually, the driving assumption in the FSDTs is that the straight lines