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Two-dimensional wake dynamics behind cylinders with triangular cross-section under incidence angle variation

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ABSTRACT

The wakes behind cylinders having an equilateral triangular cross-section are studied numerically for various cylinder inclinations and Reynolds numbers. For steady flows, the development of the recirculation region near the onset of flow separation is described, and the separation Reynolds numbers mapped for different cylinder inclinations. Cylinder inclinations that are not reflection symmetric about the horizontal centreline produce asymmetric recirculation regions which persist until the flow becomes unstable. Flow separation is observed to initiate on the rear-face of the cylinder and develops in size with increasing Reynolds numbers until the separation points become defined at the triangular cross-section's vertices where they remain even at higher Reynolds numbers. Using the Stuart–Landau equation, the critical Reynolds numbers of the different flow cases are quantified. The inclination of the cylinder is seen to strongly affect the location of the separation points, the dimensions of the recirculation region, and ultimately the critical Reynolds numbers. Increasing the Reynolds number past the instability threshold, a Bénard–von Kármán vortex street is initially observed before the downstream region of the wake re-aligns to a bi-layered vortex structure. Beyond this regime, the vortex street is observed to develop variously. At most cylinder inclinations ($\alpha < 30^\circ$ and $\alpha \gtrsim 42^\circ$), the bi-layered wake re-assembles into a secondary vortex street further downstream. For a small range of cylinder inclinations ($30^\circ \leq \alpha \lesssim 38^\circ$), the shedding vortices interact to form a vortex street similar to that produced by the 2P shedding mode for oscillating circular cylinders, while inclinations $38^\circ \lesssim \alpha < 54^\circ$ describe the development of a $P+S$-like vortex street. The formation of these unsteady wakes are attributed to vortex interactions in the wake. The drag and lift force coefficients for various cylinder inclinations and Reynolds numbers are also summarised. Phase trajectories of the force coefficients reveal that the transition from the bi-layered wake to the 2P-like wake alters its profile significantly, while the transitions to the other vortex streets observed did not incur such changes.

1. Introduction

The study of flows past bluff bodies is an ongoing area of immense research interest in fluid mechanics desiring to understand the complexities and underlying dynamics of the emerging flow structures. The wakes trailing these bodies exhibit several known features dependent on the Reynolds number, inducing different force profiles on the body. While

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most wake flow phenomena has been extensively studied for the circular cylinder (Berger and Wille, 1972; Williamson, 1996c), interest in the vortex dynamics of cylinders of prismatic cross-sections have only relatively recently received growing attention. Prismatic cross-section cylinders lack the smooth contours for flow separation that the circular cylinder possesses, and instead presents sharp corners to the flow which alters the flow dynamics. Cylinders with triangular cross-sections in particular find applications in vortex flow meters and turbulence promoters among other applications due to the sharp corners it exhibits, and are also of practical importance in the field of structural design as it models representative geometries. The present study takes a fundamental view to understand the development of these wakes at moderate Reynolds numbers prior to the introduction of complexities from turbulence and chaos.

Studies on the circular cylinder details the following: At low Reynolds numbers, the flow is steady and remains attached to the body. Taneda (1956) verified this experimentally and reported the onset of flow separation to occur at a separation Reynolds number, $Re_s=5$, exceeding which two symmetric recirculation bubbles form at the rear-side of the cylinder. More recently, Sen et al. (2009) utilised finite-element simulations to refine this separation Reynolds number as $Re_s=6.29$, and determined the recirculation length and vorticity to follow Re^1 and $Re^{0.5}$ laws, respectively. At the limit of the steady flow behaviour, the system becomes unstable via a Hopf bifurcation which is well described by the Stuart–Landau equation. The pattern of counter-rotating vortices shed alternately from the cylinder describes the Bénard–von Kármán vortex street, after Bénard (1908) and von Kármán (1911) who observed and studied the stability of the arrangements of the vortices. Jackson (1987) conducted a stability analysis on flows past a circular cylinder and reported the critical Reynolds number as 46, which various other studies found good agreement with (Mathis et al., 1984; Provansal et al., 1987; Sreenivasan et al., 1987). Further investigation into the vortex-shedding process by Perry et al. (1982) found the formation of instantaneous 'alley-ways' in the streamlines of periodic flows penetrating into an otherwise 'closed' cavity observable when the flow is steady. The instantaneous streamlines also reveal the 'centre' and 'saddle' critical points in two-dimensional incompressible flows.

A further instability in the far-wake region of two-dimensional flows has also been observed. Early observations by Taneda (1959) using an aluminium dust visualisation method, and Zdravkovich (1968, 1969) using smoke-visualization techniques elucidated this secondary vortex street. This secondary shedding is reasoned to be the manifestation of a hydrodynamic instability of the mean wake, and appears, initially, as a stationary bi-layered wake which often tends to re-arrange into a street of vortex structures of a larger scale. Durgin and Karlsson (1971), in an experiment subjecting the vortex street to a deceleration, showed that the vortex spacing is crucial to the mode of deformation of the shed vortices, and derived a criterion for eccentricities to develop in the vortices; Karasudani and Funakoshi (1994) later validated this critical vortex spacing value as 0.365 from their experiments using a circular cylinder. Smoke-wire visualizations and measurements conducted by Cimbala et al. (1988) later demonstrated the rapid spatial decay of the Bénard–von Kármán vortex street and the subsequent selective amplification of lower frequency structures in the secondary vortex street. The lower frequency structures were shown to be non-integer factors of the near-wake shedding frequency, and is thus not an instability caused by the amalgamation of the primary vortices. Further studies on the secondary vortex street by Vorobieff et al. (2002), Johnson et al. (2004) and Kumar and Mittal (2012) found the downstream distance of the onset of these structures to agree to a $Re^{-0.5}$ law, and suggests that the development of these vortical structures arise from a convective instability of the time-mean wake.

For other variously shaped bodies, the developing wake region for a two-dimensional flow, while qualitatively similar, exhibits locally different topologies—the presence of eccentricities or sharp edges on a bluff-body alters the flow dynamics sufficiently to give rise to these differences. Jackson (1987) conducted a stability analysis for various cylinder cross-section geometries to determine the effect of geometry on the onset of the periodic flow phenomena, determining the critical Reynolds numbers and corresponding critical Strouhal numbers for the different geometries tested. Sheard et al. (2003), in a study of flows past rings, showed some features of the wake to develop differently than those observed for the canonical circular cylinder where the near wake describes a lack of local symmetry at lower aspect ratios. The separation and critical Reynolds numbers for ellipses of various aspect ratios and inclinations were determined by Paul et al. (2014). A recent study by Thompson et al. (2014) investigating the stability of the wakes of elliptical cylinders showed that the secondary vortex street became increasingly complex with decreasing aspect ratios as the geometry tended to a normal flat plate. They related the behaviour of the increasingly complex vortex street to the circulation per shedding cycle introduced into the wake, the value of which increased with decreasing aspect ratio of the elliptical cylinder. The wakes of square cylinders at incidence have also shown to be an area of considerable interest. Yoon et al. (2010) conducted a parametric study on flows past inclined square cylinders to map the critical Reynolds numbers and different shedding topologies exhibited by the flow as the square cylinder inclination varies from a symmetric to an asymmetric alignment about the horizontal centreline. An investigation on the Re_s value for a square cylinder at zero incidence was conducted by Sen et al. (2011) and showed the initiation of the recirculation region to occur on the rear-face of the cylinder instead of the sharp edges.

Unconfined flows past cylinders with triangular cross-sections, however, have received noticeably less focus despite the geometry featuring sharper corners and stronger asymmetry to the oncoming flow which may alter the dynamics of the flow and the bifurcation scenarios. Most reported works on flows past these prismatic structures only focus on symmetric body orientations where the triangle apex either points directly upstream, or directly downstream. For the cylinder with its apex facing upstream, a stability analysis by Jackson (1987) reported the critical Reynolds number for the onset of unsteady flow to range within 34.318–36.370 for triangle aspect ratios of 0.8 and 1.0, respectively (35.002 by linear interpolation to an equilateral triangle aspect ratio), using a computational domain with a blockage ratio 1/10; while Zielinska and Wesfreid (1995) and De and Dalal (2006), using domains of blockage ratios 1/15 and 1/20, respectively, detailed a global mode