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The stability of a flexibly mounted rotating cylinder in turbulent annular fluid flow

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ABSTRACT

In this paper, we determine the inviscid linear stability with respect to two-dimensional disturbances of a fluid flow between two concentric cylinders. The inner rigid cylinder rotates with the angular velocity Ω_0 and is fixed on elastic hinges at each end in the transverse direction. The outer cylinder does not rotate and is rigidly fixed. We assume that the fluid flow has an inner core that rotates as a solid body with angular velocity $\Omega_0/2$ and outside the core there are turbulent boundary layers. The velocity profile of the turbulent boundary layers satisfies the viscous Camassa–Holm equations. The perturbed fluid flow is derived from Rayleigh's equation. The analysis yields an equation of motion of the cylinder equivalent to previous work without boundary layers and a basic flow of constant angular vorticity. The analysis is not restricted to a small gap between the cylinders. The results are compared with the results by Antunes et al. (1996), who consider a similar problem with uniform velocity profile and the limit of small gap. For $\rho_c/\rho_f < 1$ the results disagree in that the present analysis shows stability whereas Antunes et al. find instability. For $\rho_c/\rho_f > 1$ both theories predict stability and for larger values of ρ_c/ρ_f the agreement is good especially for small gap.

1. Introduction

Taylor–Couette flow is well known for its three-dimensional flow regimes and the existence of sufficient instability criteria of various basic flow profiles. However, the stability and the dynamic behavior of an unconstrained cylinder surrounded by Taylor–Couette flow have received less attention. A transverse motion of the cylinder yields a two-dimensional disturbance of the flow with an azimuthal dependence. The mathematical treatment of the hydrodynamic stability problem with respect to axisymmetric disturbances hence no longer applies. A normal-mode analysis of the linear inviscid stability with respect to two-dimensional rotational disturbances of circular Couette flow does not yield any sufficient criteria of instability (Drazin and Reid, 2004). A transverse disturbance motion of the cylinder however yields a two-dimensional irrotational disturbance of the fluid flow if the basic flow has constant vorticity. The stability of the motion of the cylinder is hence equivalent to the stability of the fluid flow with respect to two-dimensional disturbances. An extensive amount of work exists on stability criteria of rotors surrounded by fluid-filled clearances in swirling flow. Black (1969) investigated the effect of turbulent seals on the stability of centrifugal pumps. Fritz (1970) derived the stability of a

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Nomenclature

a	inner cylinder radius
b	outer cylinder radius
$\delta = b-a$	gap between the cylinders
$\overline{\delta} = \frac{\delta}{a}$	dimensionless gap
α	amplitude of turbulent fluctuations
$\varepsilon = \frac{\alpha}{\delta}$	dimensionless turbulent fluctuations
ψ	stream function
$U(r)$	tangential velocity of the basic flow
$\Omega(r)$	angular frequency of the basic flow
$Z(r) = \frac{1}{r}\frac{d}{dr}(r^2\Omega(r))$	vorticity of the basic flow
ω	frequency of the perturbation
Ω_0	angular frequency of inner cylinder
$\overline{\varpi} = \frac{\omega}{\Omega_0}$	normalized frequency of perturbation
$\zeta = \frac{r-a}{\delta}$	dimensionless radial coordinate
$\overline{\zeta} = \frac{\zeta}{\varepsilon}$	dimensionless boundary layer coordinate
$\overline{z} = \frac{1-\zeta}{\varepsilon}$	dimensionless boundary layer coordinate
$\omega_0 = 1/2-\omega$	dimensionless frequency
$\hat{\phi}^a_{1,2}$	stream function for boundary layer solutions close to $r=a$
$\hat{\phi}^b_{1,2}$	stream function for boundary layer solutions close to $r=b$
$\tilde{p}$	perturbed pressure
ρ_f	density of the fluid
ρ_c	density of the inner cylinder
X	displacement of cylinder in the x-direction
Y	displacement of the cylinder in the y-direction
t^*	dimensionless time unit
M_a	added mass coefficient
G	gyroscopic coefficient
K	stiffness coefficient of fluid
M_c	mass of the rotor
ω_c	frequency of the rotor without fluid
$K_c = M_c\omega_c^2$	rotor stiffness coefficient
$\lambda = \frac{\delta_c}{\delta_f}$	ratio of density of rotor and fluid
$\overline{M}_a = \frac{M_a}{\rho_f \pi a^2}$	dimensionless added mass coefficient

rotor in confined turbulent and vortex flow intended for application to induction motors. Axisa and Antunes (1992) considered the stability of a rotor immersed in an annular turbulent flow with a small gap. Their work was later extended to the stability eccentric position of the rotor (Antunes et al., 1996). Common to these works is that they are based on bulk-flow models in which the velocity is assumed to be constant across the gap. These works are based on bulk-flow models in which the tangential velocity of the basic flow is equal to an averaged velocity across the gap. Therefore bulk-flow models cannot capture the effect of pressure build-up across the gap that sustains the flow in a circular path. These models yield a rotating motion-induced inertia fluid force leading to the concept of hydrodynamic mass. For the general case without the assumption of a small gap there are two additional forces. One of the forces is equal in magnitude and opposite in sign to the fictitious Coriolis force of the hydrodynamic mass and the other is equal to the fictitious centrifugal force of the hydrodynamic mass. Jansson (2014) discusses why these additional inertial forces should not be confused with any effects of the centrifugal force and the vorticity of the fluid flow. If the gap width, $\delta = b-a$, compared to the average radius of the annulus, $R = (a+b)/2$ is much less than unity, i.e. $\delta/R << 1$, the motion-induced forces that stems from these effects are an order of magnitude smaller than the inertia force of the hydrodynamic mass.

Brennen (1976) derived the fluid force acting on a whirling cylinder in annular flow of a large gap. However, his analysis does not include any stability criteria of the cylinder. Jansson et al. (2012) derived stability criteria of a cylinder surrounded by a confined swirling irrotational flow, which included the effect of the gap width. Jansson and Åkerstedt (submitted to publication) extended the analysis to a basic flow of constant vorticity. They concluded that a swirling flow with a non-uniform radial distribution of the angular velocity, i.e. if the fluid does not rotate as a rigid body, can withhold a denser cylinder in a concentric position without being slung towards the outer cylinder, whereas a flow with constant angular

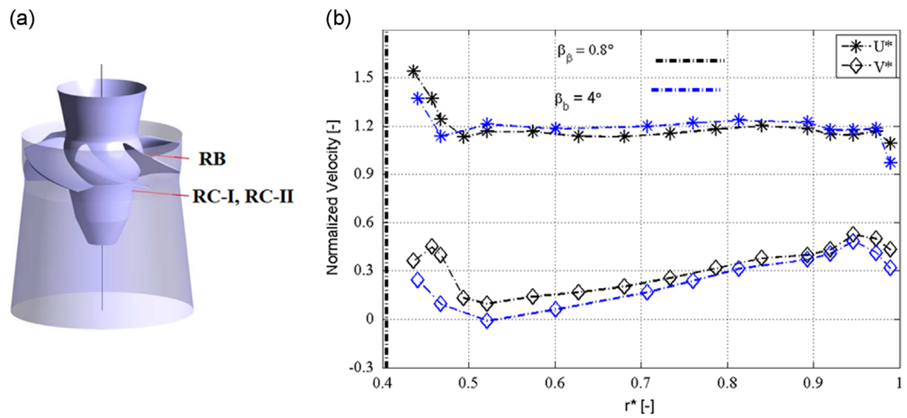

Fig. 1. (a) Geometry of the Kaplan turbine. (b) Time-averaged axial and tangential velocity profile at different sections RC-I, RC-II with two different blade angle (Kaveh Amiri licentiate, 2014 pp 86). Here V is the tangential velocity.