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Transient analytical solution for the motion of a vibrating cylinder in the Stokes regime using Laplace transforms

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ABSTRACT

A new analytical solution for the motion of an elastic cylinder in a viscous fluid is derived using Laplace transforms. Unlike previously available solutions, full expressions for transient terms are given. The solution is compared with conventional treatments of this problem. It is expected to have particular value for applications related to viscosity measurement using vibrating-wire viscometers applied to higher viscosity fluids.

1. Introduction

Vibration of cylinders has importance on scales ranging from civil engineering structures (Lu et al., 2013), to viscosity measurement (Tough et al., 1964; Padua et al., 1998; Ciotta and Trusler, 2010; Sullivan et al., 2009; Assael and Mylona, 2013; Caetano et al., 2005; Etchart et al., 2007; Harrison et al., 2007; Correia da Mata et al., 2009), micro-electro-mechanical devices (Shiraishi et al., 2013) and ultimately to studies on the behavior of nanowires (Yengejeh et al., 2014). Drag forces on arrays of cylinders are also of interest (Lu et al., 2014). In the field of microbiology, recent advances in Stokes-regime analysis have enabled realistic predictions of the motions of tiny insects and micro-organisms (Barta, 2011; Clarke et al., 2006). For very small Reynolds numbers, the equations of fluid mechanics can be linearized and exact solutions for the transient motion of the cylinder can be derived thanks to the ingenious treatment of the fluid mechanics problem by Stokes (1922). This has lead to the development of a very successful technique for measuring fluid viscosity using an instrument known as a vibrating wire viscometer (Tough et al., 1964). During one mode of operation of this instrument, a fine wire immersed in a fluid sample is set in motion using an electric circuit and the transient decay of the vibration is used to determine the fluid viscosity via an analytical solution to the problem (Retsina et al., 1987). Very accurate viscosity measurements can be obtained if the instrument is designed in such a way that it meets constraints due to assumptions used in the underlying theoretical basis for the technique (Retsina et al., 1987). Unfortunately, the constraints place limits the range of viscosity that can be measured accurately with any given instrument. Also, it can be difficult to use the technique for analysis of fluids with viscosities higher than the order of 100 mPa s, although recently significant progress has been made with higher viscosity fluids through careful design choices (Assael and Mylona, 2013; Caetano et al., 2005; Etchart et al., 2007; Harrison et al., 2007; Correia da Mata et al., 2009). One assumption that is of concern for more viscous fluids is the neglect of the initial transient forces associated with the start-up of motion. This is the subject of the present article.

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Nomenclature

a	radius of wire
B	magnetic flux intensity
D_0	damping due to internal friction
E	Young's modulus
G	function defined by Eq. (53)
I	second moment of area
L	length of wire
p	pressure in fluid
q_j	function defined by Eq. (58)
Q_j	function of the jth *eigenvalue* defined by Eq. (15)
s	Laplace transform variable
t	time
T	tension in wire
U_j	coefficient of *eigenfunction* for defining velocity
V	output voltage
v_r	velocity component in radial direction in fluid
v_θ	velocity component in tangential direction in fluid
x	coordinate position from one end of wire
y	displacement of wire
y_j	coefficient of *eigenfunction* for defining initial condition

Greek

β_1	function defined by Eq. (13a)
β_2	function defined by Eq. (13b)
θ	angular coordinate
λ_j	*eigenvalue* for mode j
μ	absolute viscosity of fluid
ν	kinematic viscosity of fluid
ρ	density of fluid
ρ_w	density of wire
ψ_j	*eigenfunction*
χ	stream function
ω_j	angular frequency for mode j

Most theoretical treatments of a vibrating cylinder in an infinite medium use the 'added mass and damping term' approach (Stokes, 1922; Retsina et al., 1987; Hussey and Vujacic, 1967; Chen et al., 1976; Mostert et al., 1989; Diller and Van der Gulik, 1991). Analysis of the flow field is considered separately from the solid mechanics in order to yield two drag terms – one proportional to acceleration (added mass term) and another proportional to velocity (damping term). These two terms are then included in the vibration problem for the rod or wire in order to represent drag from the fluid. Another possible approach for problems in the Stokes regime is to treat the problem as a linear combination of fundamental singularity solutions (Barta, 2011; Clarke et al., 2006; Yang and Hong, 1988). In this article a different approach is taken where the solid mechanics equation is solved simultaneously with the linearized fluid mechanics equations using Laplace transforms. The merit of this alternative approach is that it yields full expressions for all transient terms which hitherto have been neglected. Moreover, unlike methods which make use of fundamental stationary-oscillation solutions, the simplest initial condition to implement for the Laplace transform is the zero initial velocity distribution making it ideal for investigation of transient start-up effects.

It should be expected that transient effects will be largest for fluids of high viscosity where 'ring-down' vibrations (i.e. free damped vibrations) are being considered. In the case of steady oscillations (Brushi and Santini, 1975) the transient terms are zero by definition. Stokes (1922) argued (rightly) that it is justifiable to neglect the transient start up effects during the transient decay of free oscillations if the amplitude of the oscillation is decreasing slowly. Hussey and Vujacic (1967) and Retsina et al. (1987) showed that with a small modification to the steady oscillation solution, the effect of the log decrement on 'ideal' damped harmonic motion could be accounted for. For vibrating wire viscometers, Retsina et al. (1987) went further by proposing an analytical form for the remaining transient terms and then arguing that the terms could be neglected for the purposes of determining fluid viscosity 'if they are uniformly small', or 'if they decay rapidly compared with the decay of the main oscillation' or alternatively 'if they decay very slowly compared with the main oscillation'. To date, to the author's knowledge this assumption has only been justified experimentally – in that acceptable viscosity measurements have been achieved with both 'ring down' and forced oscillation viscometers. The transient terms can be reduced further experimentally by exciting only one mode prior to observing the decay of the free oscillations (Mostert et al., 1989). This case will also be considered.

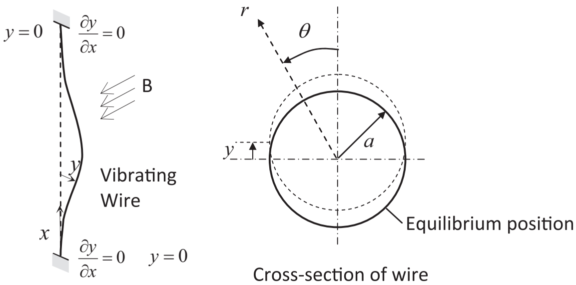

Fig. 1. Geometrical description of vibrating wire.