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The use of dynamic basis functions in proper orthogonal decomposition

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ABSTRACT

Currently, the usefulness of proper orthogonal decomposition (POD) is limited to computational domains with fixed meshes and fixed boundaries. This paper presents a new POD method that enables the modeling of flow through computational domains with deforming meshes and/or moving boundaries. To achieve this goal, the solution is approximated using basis functions which, although not explicitly functions of time, depend on parameters associated with flow unsteadiness. Results are shown for transonic flow through the Tenth Standard Configuration. Comparisons are made between this method and the standard approach for on- and off-reference flow conditions. This method properly captured flow nonlinearities and shock motion for cases in which the classical POD method failed.

1. Introduction

Reduced-order modeling has been shown to provide high-fidelity results for a wide range of applications covering transport phenomena and structural dynamics (Dowell and Hall, 2001). Through model reduction, dominant spatial modes are used to describe the flow. Using Galerkin projection, the nonlinear partial differential equations are then reduced to ordinary differential equations from which the time coefficients that weight the spatial modes are calculated.

Reduced-order modeling based on proper orthogonal decomposition (POD) has proven to be a successful method for reducing the substantial computational cost associated with high-fidelity computational fluid dynamics simulations. Proper orthogonal decomposition is a method through which snapshots of the flow obtained from the full-order model (FOM) are used to extract the optimal set of spatially dependent basis functions (Holmes et al., 1996). The large set of partial differential equations is then projected onto the basis functions, resulting in a much smaller set of ordinary differential equations.

Reviews of POD-based ROMs have been presented in Dowell and Tang (2003), Lucia et al. (2004) and Barone and Payne (2005). In the last decade, three main research directions were explored for POD-based ROMs: (i) improving the prediction of off-reference conditions, (ii) improving performance, and (iii) modeling moving/deforming meshes.

Proposed modifications to the POD basis functions to account for off-reference conditions include direct interpolation, enriching the snapshot database (Schmit and Glauser, 2004), interpolation using subspace angles (Lieu and Lesoinne, 2004; Lieu et al., 2006; Lieu and Farhat, 2007) or a tangent space to a Grassmann manifold (Amsallem and Farhat, 2008; Amsallem,

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2010; Freno et al., 2013), sensitivity analysis using parametric derivatives (Hay et al., 2008, 2010), and using actuation modes (Kasnakoğlu et al., 2008; Bourguet et al., 2011). Some of these methods are reviewed in Vetrano et al. (2011).

To improve performance for compressible flows, the use of physically or numerically sensible inner products has been suggested to better account for dynamically significant variables (Rowley et al., 2004) and to improve ROM stability (Barone et al., 2009). For multiphase flows, Brenner et al. (2012) showed that treating field variables separately when assembling the autocorrelation matrix, which yields the POD basis functions, produces greater error than using a coupled approach. To model flows with discontinuities, an augmented POD method (Brenner et al., 2010) was developed using mathematical morphology. Several acceleration techniques that reduced the computational time of two-phase flows by a factor of 114 were proposed in Cizmas et al. (2008).

The modeling of moving/deforming meshes has been primarily motivated by aeroelastic applications, which are notorious for requiring large computational resources. POD has been used in linear (Hall et al., 2000; Thomas et al., 2003; Lieu et al., 2006; Amsallem and Farhat, 2008; Bui-Thanh et al., 2008) and nonlinear aeroelastic simulations (Anttonen, 2001; Lewin and Haj-Hariri, 2005; Anttonen et al., 2005; Placzek et al., 2011; Freno and Cizmas, 2014). One of the primary challenges associated with nonlinear aeroelastic simulations is the motion of the mesh, particularly when it is deformed. Spatial and temporal integration no longer commute when the mesh varies in time. However, if the mesh is deformed in a topologically consistent manner, the integrals can commute if a computational index-based domain is used.

Anttonen (2001) and Anttonen et al. (2003, 2005) proposed using different sets of index-based basis functions associated with different deformations for potential flows; however, discontinuously changing basis functions with respect to time reduces the solution fidelity. Additionally, several sets of basis functions are required to yield a robust model, and a matching algorithm is necessary to determine the most appropriate set.

Liberge and Hamdouni (2010) used interpolation by treating the fluid–structure domain as a multiphase flow. In addition to requiring interpolation, modifications to the boundary conditions are required. Lewin and Haj-Hariri (2005) modeled the incompressible Navier–Stokes equations by using the reference frame of the moving airfoil to exploit the simplified boundary conditions that arise from incompressible viscous flow. Placzek et al. (2011) modeled compressible flow for rigid-body motions. None of these approaches address mesh deformation.

This paper presents a recently introduced POD method (Freno, 2013; Freno and Cizmas, 2014) that uses an index-based dynamic average and dynamic basis functions to model compressible flow using a deforming mesh. There is no need for interpolation or modification of the boundary conditions. These dynamic functions vary continuously with respect to parameters associated with the flow unsteadiness and/or mesh deformation, and they are optimal, subject to the prescribed form. Furthermore, only one set of basis functions is used, and therefore a matching algorithm is unnecessary. Additionally, interpolation on a tangent space to a Grassmann manifold (Amsallem and Farhat, 2008; Amsallem, 2010) is used to model off-reference flow conditions.

The proposed POD method is presented in Section 2. The dynamic average and dynamic basis functions are discussed, as well as interpolation between basis functions. The flow solver is described in Section 3. In Section 4, results are shown for transonic flow through the Tenth Standard Configuration. Comparisons are made between the full-order model and the reduced-order model using static and dynamic functions for on- and off-reference flow conditions. Additionally, the effect of the number of parameters on which the functions depend is explored. Finally, the computational savings are discussed in Section 5.

2. Proper orthogonal decomposition

Proper orthogonal decomposition is a method through which an optimal set of orthogonal spatial basis functions is extracted from a set of data, from which the mean has typically been subtracted. The spatial basis functions are linearly combined using time-dependent coefficients to form a reduced-order model:

$$\mathbf{U}(\mathbf{x},t) \approx \overline{\mathbf{U}}(\mathbf{x}) + \sum_{j=1}^{m} a_j(t)\boldsymbol{\varphi}_j(\mathbf{x}). \tag{1}$$

In (1), $\overline{\mathbf{U}}(\mathbf{x})$ is the time average, $a_j(t)$ are the time coefficients, and $\boldsymbol{\varphi}_j(\mathbf{x})$ are the basis functions. Through reduced-order modeling, the partial differential equations are reduced to a system of ordinary differential equations.

In this paper, modifications are made to POD in which the static average and static basis are replaced with a dynamic average and dynamic basis functions. The dynamic average and dynamic basis functions do not explicitly depend on time but rather on parameters $\boldsymbol{\Gamma} \equiv \{\gamma_1, \ldots, \gamma_d\}^T$ associated with the flow unsteadiness and/or mesh deformation.

The dynamic functions, namely, the dynamic average and dynamic basis functions, take the form

$$f(\mathbf{x};\boldsymbol{\Gamma}) = f_0(\mathbf{x}) + \sum_{k=1}^{d} \gamma_k f_k(\mathbf{x}).$$

The elements of $\boldsymbol{\Gamma}$ can consist of time derivatives, powers, and products of the measured quantities, provided all elements are linearly independent.

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