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Asymptotic model-based estimation of a wake oscillator for a tethered sphere in uniform flow

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ABSTRACT

We derive and investigate the nonlinear dynamics of a tethered sphere in uniform flow. A Lagrangian based model augmented by wake oscillators for the streamwise vertical and transverse directions enables model-based estimation of both structural and aeroelastic parameters via asymptotic analysis of internal resonance conditions between the transverse wake frequency and its structural counterpart. Validation of the proposed methodology is demonstrated by comparison of results with those of a benchmark experiment for a light sphere in water conducted by Govardhan and Williamson (1997) and by analysis of a spherical aerostat experiment conducted by Coulomb-Pontbriand and Nahon (2009). A numerical bifurcation analysis of the validated system reveals existence of possible quasiperiodic and non-stationary solutions that are consistent with documented instabilities of aerostats in severe environmental conditions and shed light on control mechanisms required for suppression of finite amplitude limit-cycle oscillations.

1. Introduction

Tethered aerostats are constrained lighter-than-air systems which have been widely used for remote sensing purposes for over a century. Recently, tethered aerostats have been proposed for capturing wind (Vermillion et al., 2014) and solar energy (Aglietti et al., 2008). Additional examples of tethered balloon systems have been proposed for calibration of structural parameters of a system designed for Venusian atmosphere exploration (Hall et al., 2008) and for geoengineering purposes (Davidson et al., 2012).

In light of the wide range of applications, there have been efforts to analyze the aerostat system stability (DeLaurier, 1972, 1981; Dietl et al., 2011) and to estimate its aerodynamic forces (Jones and DeLaurier, 1983). Theoretical lumped-mass models have been primarily employed for system response prediction (Coulomb-Pontbriand and Nahon, 2009; Rajani et al., 2010; Redi et al., 2011; Kang and Lee, 2009) or control design (Nahon et al., 2002), whereas continuum based models were also developed (Kim and Perkins, 2002; Stanney and Rahn, 2006) to capture the multimode response of cable dynamics. Related numerical (Behara et al., 2011; Lee et al., 2013) and experimental (Lee et al., 2013; Williamson and Govardhan, 2004; Govardhan and Williamson, 2005; van Hout et al., 2010) research that highlight the abundance of flow-induced instabilities of tethered spheres in uniform flow reveal a complex bifurcation structure that includes different types of coexisting periodic responses (Govardhan and Williamson, 1997) and intermittent or non-stationary structural dynamics (Jauvtis et al., 2001; Eshbal et al., 1997). Similar behavior was also observed in a small-scale field experiment of a spherical aerostat subject to non-stationary wind conditions (Coulomb-Pontbriand and Nahon, 2009).

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To date, there is no documentation of a comprehensive and validated theoretical model that enables prediction of the onset of vortex-induced vibration (VIV) for tethered spheres or aerostats. Thus, the objective of this research is to derive and investigate a phenomenological low-order dynamical system model for a tethered spherical aerostat that still captures the VIV stability threshold for the onset of self-excited limit-cycle oscillation.

This paper is organized as follows. Following an introduction in Section 1, we formulate in Section 2 a nonlinear dynamical system including a Lagrangian based derivation of the tethered spherical aerostat augmented by a nonlinear wake oscillator model, which incorporates documented mechanisms required to predict the VIV threshold. In Section 3, we conduct an equilibrium analysis to determine conditions for the unknown linear system parameters. In Section 4, we use the asymptotic multiple-scales method to estimate the unknown nonlinear system parameters. The resulting system parameters are validated by comparison of system response with the transverse limit-cycle amplitudes obtained from a benchmark experiment (Govardhan and Williamson, 1997). In Section 5, we numerically investigate the proposed model tailored to describe conditions of a small scale spherical aerostat model (Coulomb-Pontbriand and Nahon, 2009). We conclude with some closing remarks in Section 6.

2. Problem formulation

We derive the equations of motion for a strongly nonlinear tethered sphere (Fig. 1) using a Lagrangian approach (Meirovitch, 1988). The dynamical system is completed by a wake oscillator model that incorporates both a van der Pol term (Hartlen and Currie, 1970; Blevins, 2001) to ensure self-excited limit-cycle oscillations (LCO) beyond the flutter (or Hopf) threshold and quadratic interaction terms proposed and validated by Kim and Perkins (2002) for VIV of cable dynamics. The latter was recently used to describe the nonlinear VIV of a spherical mathematical pendulum in uniform flow (Shoshani and Gottlieb, 2011).

2.1. The elastic pendulum

Following Gottlieb (1997), the kinetic and potential energies of a tethered aerostat system defined for a reference frame located at the static equilibrium position (see Fig. 1) are

$$\begin{aligned} \mathrm{KE} &= \frac{1}{2} m \left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2 \right) \\ \mathrm{PE} &= mgZ + \frac{1}{2} k \left(\sqrt{X^2 + Y^2 + (l+Z)^2} - l_0 \right)^2, \end{aligned} \tag{1}$$

where g is acceleration of gravity, k is elastic spring stiffness, l_0 is spring free length and l is the equilibrium length defined by $l = l_0 + (\rho_F V - m)g/k$. The equations of motion for the tethered system are derived from the Lagrangian function $\mathcal{L} = \mathrm{KE} - \mathrm{PE}$:

$$m \begin{pmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{pmatrix} + E \begin{pmatrix} X \\ Y \\ l+Z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} = \begin{pmatrix} Q_X \\ Q_Y \\ Q_Z \end{pmatrix}, \tag{2}$$

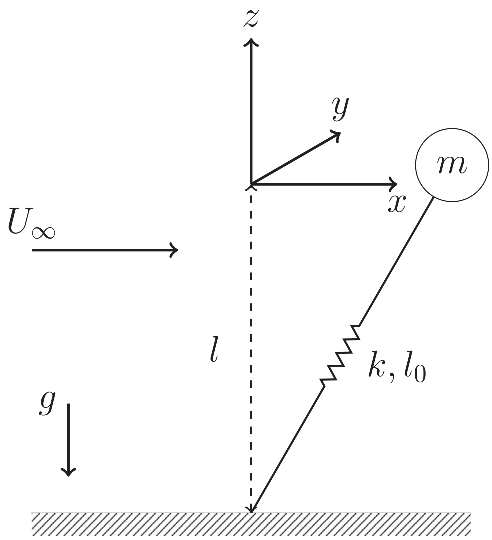

Fig. 1. Definition sketch of a tethered sphere.