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A nonlinear analysis framework for bluff-body aerodynamics: A Volterra representation of the solution of Navier-Stokes equations

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ABSTRACT

A nonlinear analysis framework for bluff-body aerodynamics based on Volterra theory is introduced to capture the linear and nonlinear aerodynamic effects. The Volterra kernels based on the impulse function concept are identified by way of the simulation of Navier-Stokes equations using computational fluid dynamics (CFD). The computational schemes used here are validated through theoretical consideration, i.e., Blasius solution for the steady-state and Theodorsen solution for the system dynamic-state simulation. The source of nonlinearities in the aerodynamics of bluff bodies is systematically investigated. The simulation of bluff-body aerodynamics based on the Volterra reduced-order modeling scheme is obtained by the convolution of the identified kernels with the external inputs, e.g., turbulent inflow or body motion for aerodynamic or aeroelastic response, respectively. It is demonstrated that the Volterra theory-based nonlinear analysis framework for bluff-body aerodynamics combined with the identification of kernels using CFD promises to capture the salient features of bluff-body aerodynamics and offers an accurate reduced-order approximation of the Navier-Stokes equations with reduced level of computational effort.

1. Background

A physically realizable dynamic system could be generally represented as in Fig. 1(a), where $x(t)$ denotes the input signal and $y(t)$ the output signal. The simplest input-output relationship is linear and any mapping operator beyond the linear regime could be treated as nonlinear relationship (Bendat and Piersol, 1980). Hence, the dynamic system could be considered as a summation of linear and nonlinear mapping operators, as shown in Fig. 1(b). Besides, any outside disturbances could be represented as an exterior noise $n_e(t)$, which is usually assumed to be uncorrelated with the output of the dynamic system. For many engineering applications, the dynamic system is ordinarily approximated by linear mapping operators based on the premise that the error introduced is acceptable. In the case of bridge aerodynamics, the aerodynamic transfer functions (aerodynamic admittances) and aeroelastic transfer functions (flutter derivatives) are traditionally utilized to characterize the dynamic relationships between gusts (inputs) and forces on the bridge deck (outputs), and

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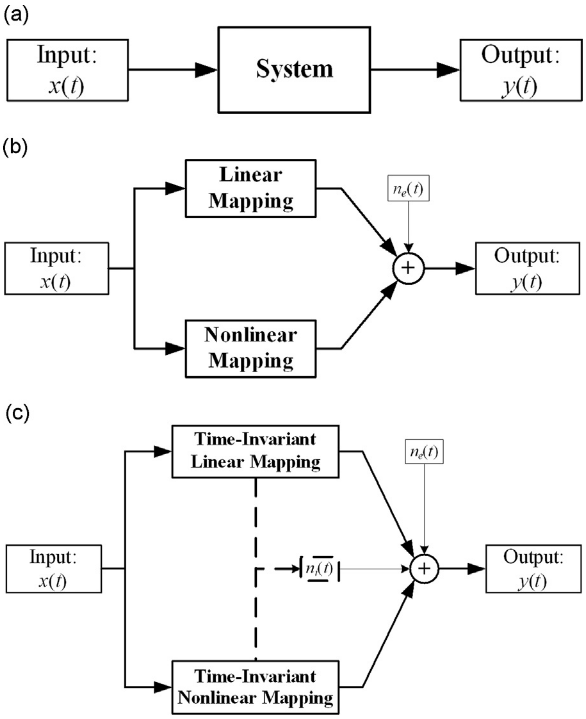

Fig. 1. Schematic representation of a dynamic system (a) A general dynamic system, (b) The summation of general linear and nonlinear mapping operators, (c) The summation of time-invariant linear and nonlinear mapping operators.

between bridge deck motions (inputs) and forces on the bridge deck (outputs), respectively (Davenport, 1962; Scanlan and Tomko, 1971). Correspondingly, both the aerodynamic admittance and flutter derivatives are linear mapping operators in the frequency domain. Since the instantaneous output $y(t)$ depends on the input $x(t)$ and its history, these linear mapping operators are functions of frequency which ensures modeling of the system memory effects. The corresponding time domain equivalents of these transfer functions are also utilized to simulate bridge aerodynamics based on a linear convolution scheme (Scanlan et al., 1974).

Rapid increase in the bridge spans and the attendant innovative bridge deck cross-sections demand that the bridge aerodynamic system may not be represented by linear mapping operators (Diana et al., 2010). Recent observations in wind tunnels have highlighted various nonlinear phenomena in bridge aerodynamics such as: (i). non-proportional relationship between input and output; (ii). multiple frequencies excited by a single-frequency; (iii). amplitude dependence of aerodynamic and aeroelastic forces and (iv). hysteretic feature of aerodynamic/aeroelastic behavior versus the angle of attack (Wu and Kareem, 2013a). In order to capture nonlinear bridge aerodynamics, a number of studies have been focused on identifying the sources of nonlinearity and their modeling (e.g., Diana et al., 1995; Chen and Kareem, 2003; Wu and Kareem, 2011a). In general, these nonlinear models cannot fully characterize nonlinear bridge aerodynamics (Wu and Kareem, 2012a). Another promising approach recently developed to consider nonlinear bridge aerodynamics is based on the Volterra theory (e.g., Wu and Kareem, 2011b, 2012b, 2012c, 2013b), where the linear convolution scheme is logically extended to the summation of linear and nonlinear convolution schemes. In such a case, the response $y(t)$ under an arbitrary input $x(t)$ may be represented as (Rugh, 1981)

$$y(t) = h_0 + \int_0^t h_1(t-\tau)x(\tau)d\tau + \sum_{n=2}^{\infty} \int_0^t \cdots \int_0^t \int_0^t h_n(t-\tau_1, \ldots, t-\tau_n)x(\tau_1)\ldots x(\tau_n)d\tau_1 \ldots d\tau_n \tag{1}$$

where h_0 represents steady-state term which satisfies system initial condition; h_1 represents the first-order kernel which describes the linear behavior of system; and h_n the higher-order terms representing nonlinear features of the system. Due to the fading memory characteristic of many dynamical systems, including bridge aerodynamics, a truncated Volterra system could be utilized. In this study, the second-order Volterra representation ($n=2$) is employed as a first attempt beyond linearity in the simulation of wind-structure interactions.

The traditional application of Volterra theory is based on the assumption that the dynamic system involved is time invariant, otherwise several intractable revisions need to be considered (Rugh, 1981). However, a typical wind-structure interaction system governed by the Navier-Stokes equations may result in an aerodynamic force that is time variant despite a time-invariant input (either gusts or deck motions). This is a consequence of wake instability (Hopf bifurcations) (Jackson, 1987) and laminar-turbulence transition (period-doubling bifurcations) (Karniadakis and Triantafyllou, 1992). These internal disturbances could be treated as an interior noise $n_i(t)$, whose properties and correlation with the output are still not well

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