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Wave forces on a submerged horizontal plate - Part I: Theory and modelling

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ABSTRACT

This paper is concerned with the propagation of nonlinear gravity waves over a thin horizontal plate submerged in water of shallow depth. An unsteady solution of the problem is obtained by use of the theory of directed fluid-sheets for the two-dimensional motion of an incompressible and inviscid fluid. Particular attention is paid to the calculation of the nonlinear wave-induced vertical and horizontal forces and overturning moment by solving the Level I Green–Naghdi equations. The theoretical formulation of the problem is given in this paper (Part I), while the results due to solitary and cnoidal waves, and comparisons with the available experimental data are given in a companion paper under the same title (Part II).

1. Introduction

Coastal bridges, piers, jetties and docks are among the structures which are exposed to open water in coastal areas. Under certain environmental conditions, such as severe storms and hurricanes, the decks of these structures may become fully submerged as a result of the increase in water level due to storm surge and other wave actions, as was observed in some of the recent natural disasters such as Hurricane Katrina (2005) in the south of the US. Similarly, submerged break waters are constantly under the influence of surface waves. To prevent the decks of submerged structures from failure, it is of considerable interest to develop accurate theoretical predictions of the wave-induced forces and overturning moments.

Here, we are specifically concerned with the nonlinear unsteady solution of the problem of cnoidal and solitary waves propagating over a two-dimensional, thin, submerged, horizontal plate. We consider the problem to be two-dimensional, as we assume the length of the plate (L_P, into the page) to be significantly larger than the plate width (B) in the direction of wave propagation. In addition, the plate is considered thin, which is an idealized model of a fully submerged deck, assuming that the thickness of the deck is much smaller than the length or width of the structure and the water depth.

Even when attention is confined to the two-dimensional motion of an inviscid, incompressible and homogeneous fluid, the difficulties associated with obtaining an analytical solution of the nonlinear equations of motion and the boundary conditions of the problem in time are far insurmountable. An alternative is to assume an irrotational flow subjected to linear boundary conditions, and use Laplace's equation as the governing equation of the fluid motion. Once the velocity potential is solved in the domain, the pressure distribution on the plate can be estimated from Euler's integral. The horizontal and

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vertical forces, and the overturning moment, are then calculated by integrating the dynamic pressure. Such an approach is followed by Siew and Hurley (1977) (the final equations of the forces were given by Patarapanich, 1984a) and Kojima et al. (1994). Lo and Liu (2014) studied the interaction of a solitary wave with a submerged plate by utilizing a linear long wave approximation, and dividing the domain into different regions and matching the solutions of different regions.

Alternatively, and in recent years, available computational fluid dynamics (CFD) solvers may be adopted to find an estimate for the forces on a submerged structure, similar to the approaches followed by Takaki (2001) and Xiao et al. (2010), and recently by Seiffert et al. (2014) and Hayatdavoodi et al. (2014). Several studies of interaction of periodic waves with submerged plates, such as that by Patarapanich (1984b), Brossard et al. (2009) and Liu et al. (2009), to name a few, are motivated by the application of the submerged plate as a wave breaker, i.e., measurements and discussion are on wave scattering due to a submerged plate rather than wave loads on the plate.

Yet another alternative is that employed by Green and Naghdi (1976b) and Green and Naghdi (1976a), who constructed the theory on directed fluid sheets. Green and Naghdi succeeded to obtain a fairly general theory, namely the Green–Naghdi theory of water waves (GN hereafter). The theory considers a three-dimensional body of the fluid known as the fluid sheet and satisfies the boundary conditions on the top and bottom surfaces of it exactly, and postulates the integrated conservation laws. In the general form of the theory, incompressibility is the only assumption made about the medium. In this paper we use the GN theory for the solution of the problem.

The GN theory relies on the top and bottom surfaces being continuous and smooth. In the case of any discontinuity curve in the domain, a special treatment is required, i.e., appropriate conditions at the discontinuity curves must be developed. Within the application of the GN equations to the problem of propagation of water waves over a submerged plate, such discontinuity occurs at the edges of the plate and requires the use of jump conditions demanded by the theory. In general, however, the necessity of utilizing the jump conditions in a specific unsteady problem is not given explicitly. If the bottom or top surfaces, for instance, were to have a slope of 90°, then it is necessary to use the jump conditions. When the slope of the bottom is slight, and the first three derivatives are small, one would expect the theory to be applicable without the jump conditions. Within the GN theory, Webster and Wehausen (1995) discuss that the geometric situation beyond which jump conditions are required must be obtained by numerical studies.

The GN equations have been applied to problems which involve a sudden change in the top or bottom surfaces. Almost in all these cases, the domain is divided into separate regions on the sides of the discontinuity curves (in two-dimensions), and the appropriate equations are solved in each region. The solution of the problem in different regions is then matched at the discontinuity curves by the use of the jump conditions demanded by the theory. Caulk (1976) was perhaps one of the first who utilized the theory of directed fluid sheets to a problem that involved discontinuity in the fluid sheet. The steady-state problem of the inviscid, incompressible fluid flow under a sluice gate was studied by dividing the domain into two regions, namely upwave and downwave, on the sides of the sluice gate. The theory was then applied to each region and solutions were matched at the discontinuity curve, where the sluice gate was located, by utilizing the appropriate jump conditions demanded by the theory. Similar problems were later studied by Naghdi and Vongsarnpigoon (1986a) and Naghdi and Vongsarnpigoon (1986b) by use of the theory of directed fluid sheets subjected to the jump conditions.

Naghdi and Rubin (1981a) used the restricted theory along with the jump conditions to develop a nonlinear steady-state solution for the problem of the motion of a floating boat-like body and transition to planning of a two-dimensional boat. In this case, the discontinuity in the fluid sheet was due to the presence of the boat on the top surface. They divided the domain into three separate regions and solved the appropriate equations in each region, subjected to the jump and matching conditions at the region boundaries. By use of the GN theory, similar approach was followed by Naghdi and Rubin (1981b) and Naghdi and Rubin (1984) to study the problem of an incompressible, inviscid fluid in a waterfall and the squat of a two-dimensional ship, respectively, and created discontinuity at the bottom and top surfaces.

The linearized version of the GN equations was utilized by Marshall and Naghdi (1990) to obtain analytical solutions for the transmitted and reflected waves by submerged steps or surface obstacles. Due to the sudden change in the thickness of the fluid sheet, jump conditions were utilized and the resultant forces on the obstacles due to the jumps were estimated. More recently, Xia et al. (2008) and Ertekin and Xia (2014) implemented the Level I GN equations to determine the response of a mat-type hydroelastic very large floating structure (VLFS). The domain was divided into three regions based on the top surface, and the regions were matched by use of appropriate jump conditions. Motions of the VLFS were then analyzed by solving the coupled GN and structural equations.

Our primary goal in these two papers (Part I here, and Hayatdavoodi and Ertekin, 2015 referred to as Part II) is to formulate the problem of nonlinear shallow-water wave loads on a submerged, flat (rigid) plate by use of the nonlinear GN equations, and to calculate the horizontal and vertical forces and the overturning moment on the plate. In this paper, we will first review the GN equations and the fundamental assumptions in deriving the equations and provide the final form of the Level I equations for a flat and stationary seafloor in Section 2. The problem of the fluid flow past a submerged plate may best be analyzed by dividing the domain into separate regions of flow, labeled as regions I, II, III and IV in Fig. 2, and considering the solution in each region and matching these solutions by use of the jump and matching conditions demanded by the theory. These are discussed in Section 3–4. We will also compare the results of the GN model with the linear solution of the problem in the case of periodic waves in Part (II). The linear method is briefly discussed in Section 5. In Part (II) of the paper, the plate is placed in a numerical wave tank capable of creating solitary and cnoidal waves. The GN equations are solved and as a result, the forces and moment on the submerged plate are calculated in several physical cases.