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Effect of mass ratio on free vibrations of a square cylinder at low Reynolds numbers

Subhankar Sen[a], Sanjay Mittal[b,*]

[a] Department of Mechanical Engineering, National Institute of Technology Agartala, Tripura 799 055, India
[b] Department of Aerospace Engineering, Indian Institute of Technology Kanpur, UP 208 016, India

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ABSTRACT

A stabilized space–time finite-element method is used to study the effect of oscillator mass ratio, m^* on in-line and transverse free vibrations of a rigid square cylinder at zero incidence in two-dimensions. The mass ratios considered are 1, 5, 10 and 20. The reduced natural frequency is $F_N = 14.39/\mathrm{Re}$ where Re, the Reynolds number, is based on the edge length of the square cylinder and free-stream speed. The structural damping coefficient is assigned a zero value. Results are presented for $50 \le \mathrm{Re} \le 250$. The cylinder may undergo vortex-induced vibrations (VIV) and/or galloping. It is found that the occurrence of galloping is a function of mass ratio. Galloping is not observed for the low mass ratio considered ($m^* = 1$), but strong galloping effects are realized for $m^* \ge 5$. The absence of galloping for $m^* = 1$ marks significant difference in frequency, response and force characteristics as compared to the cases of higher mass ratios. The response behaviour of $m^* = 1$ cylinder is characterized by the initial and lower branches. For $m^* \ge 5$ an additional galloping branch (Sen and Mittal, 2011. Journal of Fluids and Structures 27, 875–884) is observed. The onset of galloping is marked with the occurrence of mismatch of frequency of vortex-shedding and body oscillation. The Reynolds number or reduced speed marking the onset of lock-in increases with increasing m^*. In contrast, the Re or reduced speed for onset of galloping decreases with increase in m^* and varies as $m^{*-1.3}$. The vortex-shedding is characterized by the 2S and C(2S) modes in the VIV regime. It is 2S during galloping for low oscillation amplitude and changes to 2P+2S when the transverse displacement surpasses a threshold value (0.7D, approximately where D is the edge length of square). Weak and strong hystereses at the onset of lock-in and galloping, respectively, are displayed by the $m^* = 5$ cylinder. No hysteresis is observed for $m^* = 1$. As m^* increases, the primary hysteresis becomes stronger and secondary hysteresis disappears. The Re, at which the phase jump of $\approx 180^\circ$ between lift and transverse response occurs, is virtually independent of m^*. Unlike a freely vibrating circular cylinder where the maximum transverse response increases with decreasing m^*, the variation of oscillation amplitude with m^* for square cylinder is non-monotonic in both the lock-in and galloping zones.

1. Introduction

The unsteady/periodic aerodynamic forces and moment exerted by fluid in cross-flow on a rigid body beyond the critical Reynolds number may lead to free or self-excited vibration of the body provided it is mounted elastically and the structural

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[*] Corresponding author. Tel.: +91 512 259 7906; fax: +91 512 259 7561.
E-mail address: smittal@iitk.ac.in (S. Mittal).

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damping is sufficiently small. The vortex-induced vibrations (VIV) of a flexible body or flexibly mounted rigid body belong to fluid-structure interaction, a rapidly emerging area of overwhelming engineering significance. Galloping is a large amplitude and low frequency self-excited instability characteristic of non-circular sections above a certain threshold value of reduced speed and differs from VIV (Blevins, 1990). The study of galloping of a square cylinder is of prime importance in offshore and civil engineering (Joly et al., 2012). Various aspects of flow-induced vibrations are well documented in several overviews, such as Marris (1964), King (1977), Sarpkaya (1979, 2004), Griffin and Ramberg (1982), Bearman (1984), Parkinson (1989), Okajima (1993) and Williamson and Govardhan (2004).

Apart from oscillator cross-section, the family of non-dimensional parameters influencing fluid–structure interaction includes mass ratio, m^*; damping ratio or coefficient of structural damping, ζ; reduced speed, U^* and Reynolds number, Re. Mass ratio, i.e. the ratio of oscillator mass per unit length, m and mass of displaced fluid, is a measure of susceptibility of lightweight structures to flow-induced vibration (Blevins, 1990). For an oscillator of square cross-section, mass ratio is defined as $m^* = m/\rho D^2$ where ρ is the density of fluid and D the edge length of square. The reduced speed is defined as $U^* = U/f_N D = 1/F_N$ where U is the free-stream speed and f_N the dimensional natural/structural frequency of the oscillator (in Hz). F_N is the reduced/non-dimensional natural frequency of the oscillator. The Reynolds number is based on the edge length of cylinder and free-stream speed. Lock-in/synchronization is an essential feature of flow-induced vibrations of an obstacle. The state of lock-in for a vibrating cylinder is reached when the frequencies of transverse cylinder oscillation and vortex-shedding become identical. For a cylinder of high m^*, both of these frequencies also lock onto the natural frequency. Hence, the ratio of transverse oscillation to natural frequency or frequency ratio, f^*, attains a value close to unity (Williamson and Govardhan, 2004). A brief review of some of the earlier studies on single and two-degrees-of-freedom motion of circular, square/rectangular cylinders is given below. The focus is on the effect of mass ratio. The effect of the amplitude of forced motion on the wake modes is also described. Free vibrations including galloping instability are also reviewed. In the discussions that follow, X and Y represent the in-line and transverse displacements of the cylinder, respectively.

1.1. *Single-degree-of-freedom or Y-only motion of a circular cylinder*

The vortex-induced forces primarily lead to transverse-motion of a flexibly mounted body. Majority of the earlier studies thus concentrated on transverse-only motion of a cylinder. For certain degree of damping, Feng (1968) conducted pioneering aeroelastic experiments on free vibrations of a rigid circular cylinder of $m^* \approx 250$. The response consists of two branches and transition between the branches is hysteretic. The response-reduced velocity curves indicate that the maximum transverse amplitude occurs approximately midway of the lock-in regime. Khalak and Williamson (1999) revisited this problem and closely studied the branching behaviour of cylinder response via experiments on an elastically mounted rigid circular cylinder subjected to free transverse-only vibrations. Three response branches, namely, the initial, upper and lower, were identified for low values of the combined mass-damping parameter, $m^*\zeta$. Following the terminology of Khalak and Williamson (1999), the response branches obtained by Feng (1968) are initial and lower. For transverse-only motion, Govardhan and Williamson (2000) experimentally studied the free vibration of a circular cylinder of varying mass ratio and structural damping at high Re. For various mass ratios ranging between 1 and 50, Willden and Graham (2006) reported numerical results for transverse hydroelastic VIV of a circular cylinder for $\mathrm{Re} \leq 400$ and zero structural damping. Depending on the value of m^*, they identified three distinct regimes of cylinder response, namely, primary, secondary and tertiary. Effect of mass ratio, damping coefficient and combined mass-damping on response of a freely vibrating circular cylinder was numerically investigated by Bahmani and Akbari (2010) for Re = 80–160. They found that the decrease in mass ratio or damping coefficient is associated with increase in response and range of reduced speed for lock-in. They also noted that the oscillator system behaves non-linearly with respect to m^* and ζ.

1.2. *Two-degrees-of-freedom or X–Y-motion of a circular cylinder*

Even though the dominant structure motion is in the transverse direction, the in-line displacement is also significant when the mass ratio is not too large (Mittal and Kumar, 1999). The two-degrees-of-freedom motion, i.e. in-line and transverse vibrations, of a structure is a more general and practical case of VIV. Inclusion of the additional in-line degree-of-freedom modifies the cylinder response, fluid forces, etc. Using a stabilized space–time formulation (Tezduyar et al., 1992a, b), Mittal and Kumar (1999) investigated, at Re=325, in-line and transverse free vibrations of a circular cylinder of $m^* = 4.7273$ and $\zeta = 3.3 \times 10^{-4}$. A departure of the shedding frequency from the structural frequency for a certain range of the latter was observed and was defined as the phenomenon of ‘soft lock-in’. The phenomenon of soft lock-in is characteristic to low m^*, where the frequency of oscillation during lock-in overshadows its natural counterpart and consequently, $f^* < 1$. Jauvtis and Williamson (2004) conducted pioneering experiments on two degrees-of-freedom VIV of a circular cylinder that employed identical values of mass and natural frequency in both the X and Y directions. The mass ratio was varied from 1 to 25 and the range of Re considered was 1000–15 000. For $m^* > 6$, the maximum amplitude, response branches, fluid forces and vortex-shedding modes with Y and X–Y motion are same. In contrast, for $m^* < 6$, an additional response branch, i.e. the high amplitude super upper branch appears with $X-Y$-motion. Corresponding vortex-shedding mode is 2 T where three vortices are formed each half cycle. Depending on the grouping of shed vortices in an oscillation cycle, the identification and classification of various modes of vortex formation was introduced by Williamson and Roshko (1988) (see Section 1.3). Prasanth et al. (2011) studied numerically the effects of mass ratio (=1–100) and blockage, B

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