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A model of impeller whirl for baffled mixing vessels

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ABSTRACT

A model of impeller whirl that provides stability limits and mean square radial deflections for industrial mixing vessels is introduced. The model has a linear form and includes contributions to the mass, damping, and stiffness that arise from fluid forces acting on the impeller. The fluid forces were derived from Large Eddy Simulation (LES) of a mixer with a pitched blade impeller rotating at speed N, and undergoing prescribed circular orbits of frequency Ω. Simulations were performed for Reynolds numbers, $10 \le \mathrm{Re} \le 10\,000$, and subsynchronous whirl frequency ratios, $-1 \le \Omega/2\pi N \le 1$. Model predictions of impeller whirl instability and mean square vibration amplitude were compared to experimental data from a laboratory scale mixer. The simplicity and economy of the new model allows its use in on-line diagnostics and operational planning.

1. Introduction

The objective of the present study was the development of a mathematical model for small amplitude vibration of rotating impellers that could be used as a diagnostic and operational planning tool for mixing equipment. One requirement of the model was that it could run faster than real time and could be applied to potentially thousands of operating conditions in an economical manner. A second requirement of the model was that it provides stability limits and the mean square amplitude of the random impeller orbits directly without the need for a time domain solution. These requirements effectively rule out direct, fully coupled computational fluid–structure interaction models.

All of the above requirements can be met by a linear lumped-parameter approach of minimal dimensionality provided that the important effects of the fluid forces on the orbiting impeller can be accurately modelled using the concept of fluid added mass, damping and stiffness. Such an approach has been applied to the modelling of pump and turbine vibration as described by Jery et al. (1985) but it has not been applied to mixing equipment where the flow is highly turbulent and the impeller orbits are predominantly random. The method derives the fluid forces for a prescribed circular orbit of the impeller and projects them onto the impeller displacement, velocity and acceleration which are harmonic functions. In the present study Large Eddy Simulation (LES) was used to obtain the fluid forces on the orbiting impeller rather than experiment as used by Jery et al. (1985). An advantage of this modelling approach is that once the fluid added coefficients are determined for a given mixer geometry they may be used over the entire range of operating conditions without recourse to further time consuming CFD simulations. Using CFD to extract the fluid added coefficients requires that the level of CFD resolution be adequate to extract the pertinent features of the mixing system as well as efficient so that it can support simulations for each new impeller geometry. The present study provides initial guidance on these matters.

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Nomenclature

Uppercase

$C=C_s+C_f$ total diagonal damping, $\mathrm{N\,s\,m^{-1}}$
[**C**] damping matrix, $\mathrm{N\,s\,m^{-1}}$
C_f fluid added diagonal damping, $\mathrm{N\,s\,m^{-1}}$
C_P pressure coefficient,
C_s structural diagonal damping, $\mathrm{N\,s\,m^{-1}}$
C_T thrust coefficient
D impeller diameter, m
E modulus of elasticity, $\mathrm{N\,m^{-2}}$
$E[r^2]$ expected value of mean square impeller displacement, $\mathrm{m^2}$
F_N, F_T components of fluid forces in the orbital frame, N
$\overline{F_N}, \overline{F_T}$ average forces over one orbit period, N
$\overline{\overline{F_N}}, \overline{\overline{F_T}}$ average of the orbital average fluid forces over a number of orbits, N
F_p buckling load, N
$H(\omega)$ transfer function
I second moment of shaft cross sectional area, $\mathrm{m^4}$
$K=K_s+K_f$ total diagonal stiffness, $\mathrm{N\,m^{-1}}$
[**K**] stiffness matrix, $\mathrm{N\,m^{-1}}$
K_f fluid added diagonal stiffness, $\mathrm{N\,m^{-1}}$
K_s structural diagonal stiffness, $\mathrm{N\,m^{-1}}$
$M=M_s+M_f$ total diagonal mass, kg
[**M**] mass matrix, kg
M_f fluid added diagonal mass, kg
M_I impeller mass, kg
M_s structural diagonal mass, kg
M_{sh} shaft mass, kg
N rotational speed of impeller, $\mathrm{rev\,s^{-1}}$
N_p power number
P pressure, $\mathrm{N\,m^{-2}}$
$\overline{P}$ time-averaged pressure, $\mathrm{N\,m^{-2}}$
Re Reynolds number
$S(\omega)=S_f(\omega)+S_i(\omega)$ power spectral density of excitation forces
$S_f(\omega)$ power spectral density of fluid excitation forces
$S_i(\omega)$ power spectral density of rotating imbalance force
S_0 low frequency approximation of $S_f(\omega)$
$\forall$ impeller volume, $\mathrm{m^3}$
Y, Z components of impeller displacement in inertial frame, m
$\dot{Y}, \dot{Z}$ components of impeller velocity in inertial frame, $\mathrm{m\,s^{-1}}$
$\ddot{Y}, \ddot{Z}$ components of impeller acceleration in inertial frame, $\mathrm{m\,s^{-2}}$

Lowercase

$c=c_s+c_f$ total off-diagonal damping
c_f fluid added off-diagonal damping
c_s structural off-diagonal damping
e mass eccentricity, m
f frequency, Hz
f_X axial thrust on impeller, N
f_Y, f_Z components of the fluid forces in the horizontal plane of the inertial frame, N
$g=9.8$ gravitational acceleration, $\mathrm{m\,s^{-2}}$
$k=k_s+k_f$ total off-diagonal stiffness, $\mathrm{N\,m^{-1}}$
k_f fluid added off-diagonal stiffness, $\mathrm{N\,m^{-1}}$
k_s structural off-diagonal stiffness, $\mathrm{N\,m^{-1}}$
l shaft length, m
$m=m_s+m_f$ total off-diagonal mass, kg
t time, s

Greek symbols

Ω orbit rotational speed, $\mathrm{rad\,s^{-1}}$
Γ mesh stiffness
δ_j phase angle of the jth eigenmode, rad
ϵ impeller orbit eccentricity
θ impeller angular position, rad
λ damping exponent
μ dynamic viscosity, Pa s
μ_{eff} effective dynamic viscosity, Pa s
μ_{sgs} dynamic viscosity (subgrid scale stresses), Pa s
ρ fluid density, $\mathrm{kg\,m^{-3}}$
ω angular frequency, $\mathrm{rad\,s^{-1}}$
ω_d whirl frequency of the jth eigenvalue, $\mathrm{rad\,s^{-1}}$
ω_{nb} bending natural frequency of impeller and shaft in air, $\mathrm{rad\,s^{-1}}$

Vectors in complex plane

$\mathbf{A}=A+ia$ eigenvector of the linear model, m
$\mathbf{C}=C+ic$ complex damping
$i^2=-1$
$\mathbf{K}=K+ik$ complex stiffness
$\mathbf{M}=M+im$ complex mass
$\mathbf{s}=\lambda+i\omega$ eigenvalues of the linear model, $\mathrm{s^{-1}}$
$\mathbf{r}=X+iY$ impeller displacement vector

Superscript

$*$ quantity normalized using shaft rotation rate N
$+$ quantity normalized using the natural bending frequency of shaft in air ω_{nb}

A schematic of a baffled mixing vessel with pitched blade impeller is shown in Fig. 1. The role of the impeller is to circulate the fluid while the radial baffle plates enhance mixing. Fluid forces acting on the impeller arise from instantaneous asymmetric blade loading that results from fluid turbulence or impeller deflection.

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