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A review of rotating cylinder wake transitions

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ABSTRACT

Recent work on the flow past a rotating cylinder is reviewed and further investigated at low Reynolds numbers. The various two- and three-dimensional transitions that occur as the rotation rate is increased are detailed. Two steady states, steady state I and steady state II, are identified based on the physical characteristics of the wake and the drag force on the body. Steady state I occurs at lower rotation rates, while state steady state II occurs at higher rotation rates. Linear stability analysis shows that two three-dimensional modes become unstable on steady state I and steady state II. Floquet stability analysis of the unsteady base flows that occur at very low rotation rates shows the presence of five three-dimensional modes. The curves of marginal stability are presented, followed by a comparison of numerical simulations to their experimentally obtained counterparts. Furthermore, the spatio-temporal characteristics of each mode and the likely underlying physical mechanisms are briefly discussed.

1. Introduction

In this study, we review the recent developments in the understanding of the wake of an isolated rotating cylinder in freestream at low Reynolds numbers. The flow is a function of two parameters: the flow Reynolds number (Re) and non-dimensionalised rotation rate (α). The Reynolds number is given by $\mathrm{Re} = UD/\nu$, where U is the incoming freestream velocity, D is the cylinder diameter and ν is the kinematic viscosity of the fluid. The rotation rate is defined by $\alpha = \omega D/2U$, where ω is the angular velocity of the cylinder. The rotation rate is also equivalent to twice the ratio of the surface speed of the cylinder to the freestream velocity. In this paper, we examine the various flow transitions that occur for $\alpha < 7$ and $\mathrm{Re} \le 400$.

The wakes from a non-rotating circular cylinder in the low Reynolds number range, together with the various transitions that occur, have been extensively documented. Amongst others, a comprehensive review detailing the steady and unsteady regimes has been presented by Williamson (1996a). Unsteady flow is observed on increasing the Reynolds number past $\mathrm{Re} \simeq 47$, which is characterised by the shedding of vortices alternately from the top and bottom separating shear layers to form a vortex street. This is commonly referred to as Bénard–von Kármán shedding. On further increasing the Reynolds number, secondary three-dimensional vortices begin to form on the otherwise two-dimensional wake at $\mathrm{Re} \simeq 190$. The wake vortices develop spanwise waviness of approximately four cylinder diameters, and this state is referred to as *mode A*

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shedding (Williamson, 1988). Another three-dimensional mode, mode B, appears with a spanwise wavelength of $0.8D$ on further increasing the Reynolds number above $\text{Re}=230$–240 (Williamson, 1988; Wu et al., 1996a, 1996b, 1994). Numerical investigations by several research groups (e.g., Barkley and Henderson, 1996; Thompson et al., 1996; Zhang et al., 1995) observed the associated three-dimensional structures. Mode A is observed to grow primarily in the vortex cores, while mode B grows in the braid shear layers connecting the opposite-signed vortex cores. These two modes do not introduce any secondary frequencies in the wake, although in their saturated state they do modify the primary wake frequency slightly. The analogues of modes A and B have also been detected in the wakes of other bluff bodies, such as square cylinders (Robichaux et al., 1999) and elongated bluff bodies (Ryan et al., 2005).

Other modes which do introduce new frequencies have also been observed in the wakes of bluff bodies. These modes are quasi-periodic in nature, and the secondary frequencies can be due to modulated standing waves or due to travelling waves in the spanwise direction. Because of their quasi-periodic nature, they have been referred to as QP modes (Blackburn and Lopez, 2003; Blackburn et al., 2005).

Another three-dimensional mode can be observed when the wake symmetry is altered, as in the wakes behind tori of different aspect ratios (Sheard et al., 2004a, 2004b, 2005), inclined square cylinders (Sheard et al., 2009; Sheard, 2011), and the asymmetric wakes found behind oscillating cylinders (Leontini et al., 2007). Typically, this mode has a spanwise wavelength between modes A and B and is $2T$ periodic (i.e., subharmonic), where T is the period of vortex shedding. It is referred to as *mode C*, and was also observed for a non-rotating cylinder when a trip wire was placed in the wake (Zhang et al., 1995; Yildirim et al., 2013a, 2013b) for $165 \leq \text{Re} \leq 300$.

Cylinder rotation brings about asymmetrical wake flow, leading to a net lift force. This is commonly known as the Magnus effect (Prandtl, 1926). Flow past rotating cylinders at low Reynolds numbers has been investigated by several research groups (Mittal and Kumar, 2003; Stojković et al., 2002, 2003; Kang et al., 1999; Akoury et al., 2008; Pralits et al., 2010, 2013), who observed two distinct shedding regimes. At low rotation rates ($\alpha \lesssim 2$), the classical Bénard–von-Kármán (BvK) vortex street is observed (also known as *mode I shedding*), where vortices are shed alternatively. The second shedding regime (also known as *mode II shedding*) occurs at higher α over a narrow band. In this regime, single-sided vortex shedding occurs with a period much longer than for mode I shedding. Recent experimental investigations by Kumar et al. (2011) and Balcarová (2011) confirmed the existence of these two shedding regimes.

Very few investigations have been carried out on the development of three-dimensionality in rotating cylinder wakes. Three-dimensional DNS was performed by Akoury et al. (2008) at low rotation rates of $\alpha < 1.5$. They observed that the onset of the mode A instability was delayed to higher Reynolds numbers as the rotation rate was increased. At $\alpha=0.5$, the critical Reynolds number was predicted to be approximately 220, and at $\alpha=1.5$, $\text{Re}=200$, the flow remained two-dimensional. Mittal (2004) performed three-dimensional simulations at $\alpha=5$, $\text{Re}=200$ with different end conditions and observed small spanwise structures, which were possibly due to centrifugal instabilities. Stability analysis and three-dimensional simulations were performed by Meena et al. (2011) at higher rotation rates; these showed that the flow was unstable to perturbations for $3 \leq \alpha \leq 5$ at $\text{Re}=200$.

Recent numerical investigations by Rao et al. (2013a, 2013b) showed several three-dimensional modes becoming unstable to spanwise perturbations in the steady and unsteady regimes of flow for $\text{Re} \leq 400$. Five three-dimensional modes were found to be unstable in the mode I shedding regime, while four three-dimensional modes were observed in the steady regimes of flow for $\alpha \gtrsim 2$. The saturated state of these modes has been observed experimentally by Radi et al. (2013). The experimentally observed modes were in excellent agreement with those observed in the numerical simulations.

In this study, we review and present results from the linear stability analysis and present some comparative experimental visualisations. The remainder of this paper is organised as follows. In Section 2, the numerical formulation and the experimental setup are described, followed by the results from the stability analysis and a direct comparison to the experimental visualisations in Section 3. The conclusions summarise and provide a perspective on the findings.

2. Methodology

2.1. Numerical formulation

A spectral-element formulation was used to discretise the incompressible Navier–Stokes equations in two dimensions. The computational domain consists of quadrilateral macro-elements, which are further subdivided using internal node points distributed according to the Gauss–Legendre–Lobatto quadrature points. The velocity and pressure fields are represented by tensor products of Lagrangian polynomial interpolants. For smooth problems, as the polynomial order is increased, spectral convergence is achieved (Karniadakis and Sherwin, 2005). An unsteady solver employing a fractional time-stepping method was used to integrate the convection, pressure and diffusion terms of the Navier–Stokes equations. More details of this solver can be found in Thompson et al. (2006).

To investigate the three-dimensional stability of the flow to spanwise perturbations, linear stability analysis was employed. The linear stability equations were marched forward in time for initially random perturbations for a given wavelength. After some time, only the first few dominant amplifying or decaying modes remain. This method was used to obtain the fastest growing modes, and has previously been used to determine the onset of three-dimensionality for a variety of different problems, including oscillating cylinders (Leontini et al., 2007; Lo Jacono et al., 2010; Leontini et al., 2013) and rotating cylinders near a wall (Stewart et al., 2006, 2010; Rao et al., 2011). For a periodic base flow, the amplification factor of

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