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Nonlinear flexural waves in fluid–filled elastic channels

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A B S T R A C T

Nonlinear waves on liquid sheets between thin infinite elastic plates are studied analytically and numerically. Linear and nonlinear models are used for the elastic plates coupled to the Euler equations for the fluid. One-dimensional time-dependent equations are derived based on a long-wavelength approximation. Inertia of the elastic plates is neglected, so linear perturbations are stable. Symmetric and mixed-mode travelling waves are found with the linear plate model and symmetric travelling waves are found for the nonlinear case. Numerical simulations are employed to study the evolution in time of initial disturbances and to compare the different models used. Nonlinear effects are found to decrease the travelling wave speed compared with linear models. At sufficiently large amplitude of initial disturbances, higher order temporal oscillations induced by nonlinearity can lead to thickness of the liquid sheet approaching zero.

1. Introduction

Interactions between fluids and elastic boundaries occur frequently in both natural and mechanical environments (Korobkin et al., 2011). Such problems are mathematically challenging, due the coupling between the moving fluid and the deformable boundary. In this paper we examine the flow of an inviscid fluid between two thin elastic plates. Examples of systems where similar types of waves arise include the pulmonary system (e.g. Grotberg, 1994; Walsh, 1995), in flat-plate-type fuel assemblies used in the cooling systems of nuclear reactors (e.g. Kim and Davis, 1995) and in energy-harvesting devices (see Tang, 2007).

The linear stability of similar fluid–elastic systems has been examined by many authors in the past. Walsh (1995) used shell theory to examine pipes with three different kinds of flow; internal, annular and planar. The internal flow was axisymmetric in a flexible pipe, the annular flow was axisymmetric, between a flexible inner shell and a rigid outer one, and the planar flow considered flow between a rigid plane and a flexible one. He found that, for inviscid flow, tubes where the displacements in both the axial and normal directions are considered together are unstable to long wave flutter, but stable if each are considered on their own. Nonlinear stability of fluid-loaded elastic plates was investigated by Peake (2001). De Langre and Ouvrard (1999) considered instabilities which arise in a flexural pipe, with a spring foundation and under tension, for the coupled mode.

De Langre (2002) expanded upon this work to allow both symmetric and antisymmetric perturbations, in a paper examining regions in which convective instabilities change to absolute instabilities and vice versa. More recently,

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the linear stability of a finite number of coupled parallel flexible plates in axial flow has been studied by Michelin and Llewellyn Smith (2009). Experiments on the flutter instability of assemblies of two or more parallel flexible cantilevered plates have been performed by Schouveiler and Eloy (2009). The coupled dynamics of two parallel cantilevered flexible plates of finite length in axial flow has been studied in Tang and Païdoussis (2009). Branches of nonlinear symmetric and antisymmetric travelling waves and other bifurcation branches for fluid sheets between thin elastic plates were also computed by Blyth et al. (2011).

In the absence of elastic walls, we recover the classical problem of plane liquid sheets. Squire (1953) studied the temporal instability of a plane liquid sheet moving through air. He showed that long waves were unstable, and the antisymmetric disturbances are less unstable than the symmetric ones. He also calculated the largest growing mode by considering the maximum value of the imaginary component of the frequency. The spatial instability equivalent was studied by Li (1993) for high flow velocities through a gaseous medium. He found the dispersion relation for antisymmetric and symmetric modes for different gas-to-liquid density ratios, and analysed growth rates of instabilities for different parameter values.

The linear stability analysis of thin planar liquid sheets was extended by Mehring and Sirignano (1999) by considering the fully nonlinear problem. They examined nonlinear symmetric and antisymmetric capillary waves by reducing the two-dimensional unsteady problem to a one-dimensional unsteady problem, assuming that the liquid sheet thickness is small compared with the wavelength of the disturbance. Numerical simulations were used to study the time evolution of the waves and differences were found compared with the linear case. Travelling waves were computed in the symmetric case. They also investigated the nonlinear deformation of semi-infinite sheets, forced periodically at one end.

In this paper we extend the Mehring and Sirignano (1999) method to study nonlinear symmetric and antisymmetric waves in fluid sheets between two infinite elastic walls. A system of one-dimensional unsteady equations is derived in three cases: a linear-flow–linear-plate model, a nonlinear-flow–linear-plate model and a nonlinear-flow–nonlinear-plate model. The plates are assumed inextensible and their inertia is neglected. In the third case the plates are modelled using the special Cosserat theory of hyperelastic shells, satisfying Khirchoff's hypothesis (see also Plotnikov and Toland, 2011; Blyth et al., 2011). Travelling waves are found in each case. The evolution in time of symmetric and antisymmetric waves is compared for different problems and the nonlinear effects are discussed.

In Section 2 we outline the problem and derive the governing equations before finding solutions to the linearised system in Section 3. Travelling waves are then sought numerically for the leading order curvature equations in Section 4. In Section 5, we seek travelling wave solutions to the nonlinear system and introduce a numerical scheme to determine the nonlinear temporal evolution of the linear wave profiles calculated in Section 2 using a Crank-Nicolson scheme.

2. Formulation

Working in Cartesian coordinates (x^*, y^*), we consider an incompressible, inviscid, two-dimensional flow between two flexible walls a mean distance a apart, in a gravity-free regime (see Fig. 1). The fluid has constant density ρ, and its pressure and velocity fields are denoted p^* and (u^*, v^*) respectively. The upper interface is located at $y^* = \eta^*_+(x^*, t^*)$ and the lower at $y^* = \eta^*_-(x^*, t^*)$. Outside of the channel is assumed to be a vacuum. If the flow and walls are undisturbed, the fluid is assumed to flow at a constant velocity U_r in the x^* direction, with the walls at $y^* = \pm a/2$. We are interested in long wavelength perturbations to this undisturbed state with wavelength L. The equations governing the unsteady fluid flow are the two dimensional Euler equations:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*}, \tag{2}$$

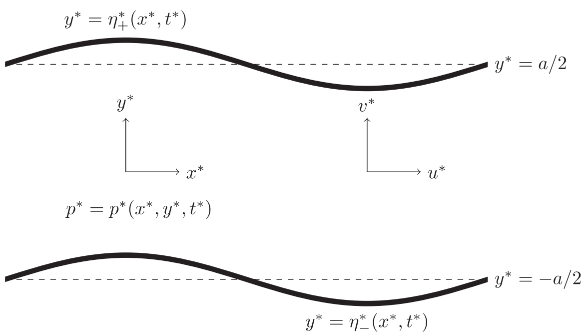

Fig. 1. Dimensional problem setup.