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Instability of sloshing motion in a vessel undergoing pivoted oscillations

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ABSTRACT

Suspending a rectangular vessel partially filled with an inviscid fluid from a single rigid pivoting rod produces an interesting physical model for investigating the dynamic coupling between the fluid and vessel motion. The fluid motion is governed by the Euler equations relative to the moving frame of the vessel, and the vessel motion is given by a modified forced pendulum equation. The fully nonlinear, two-dimensional, equations of motion are derived and linearised for small-amplitude vessel and free-surface motions, and the natural frequencies of the system analysed. It is found that the linear problem exhibits an unstable solution if the rod length is shorter than a critical length which depends on the length of the vessel, the fluid height and the ratio of the fluid and vessel masses. In addition, we identify the existence of 1:1 resonances in the system where the symmetric sloshing modes oscillate with the same frequency as the coupled fluid/vessel motion. The implications of instability and resonance on the nonlinear problem are also briefly discussed.

1. Introduction

The movement of a vessel partially filled with a fluid can cause the fluid motion to undergo extremely complex motions. Moreover, the fluid motion, comprising waves sloshing back and forth along the fluid free-surface, produces forces and moments on the vessel, which if the vessel is free to move, can cause unintended vessel motion which could be stabilizing or destabilizing. A simple example of such unintended motion can be found in the article 'walking with coffee' which examines the spilling of coffee while walking (Mayer and Krechetnikov, 2012). A more dramatic example of a destabilizing fluid motion is the dynamics of trapped seawater on the deck of Alaskan king crab boats. They have been observed to capsize when the trapped water sloshes backwards and forwards creating unintended moments enhancing the roll motion of the boat (Dillingham, 1981; Caglayan and Storch, 1982; Adee and Caglayan, 1982). Therefore, the ability to identify destabilizing motions in coupled fluid–vessel interactions is of great practical importance. Examples where this coupling is important are terrestrial and maritime fluid transportation, space transport, fuel tanks under earthquake excitement and industrial applications such as tuned liquid dampers (TLDs) (Ikeda and Nakagawa, 1997; Frandsen, 2005).

Studying the motion of a fluid in a stationary or forced vessel is already very complicated both experimentally and theoretically. The works by Moiseyev and Rumyantsev (1968), Ibrahim (2005) and Faltinsen and Timokha (2009), and the references herein, highlight the problems in these areas. The problem of coupled dynamics adds an additional layer of

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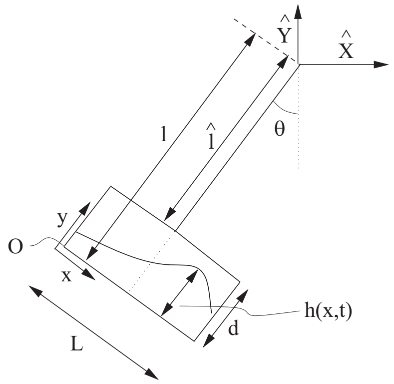

Fig. 1. Pendulum vessel experiment under consideration.

complexity to this problem because it allows for the potential enhancement or destabilisation of fluid dynamics due to the motion of the vessel.

The configuration of interest in this paper is shown in Fig. 1. The vessel, with rectangular cross section of length L and height d, is suspended by a rigid rod of length $\hat{l}$ which is attached to the top of the vessel and is free to rotate in the vertical plane such that the rod makes an angle θ with the downward vertical. The values of $\hat{l}$ and d are only important in the combination $l=\hat{l}+d$, which is the perpendicular distance from the pivot point to the base of the vessel. The vessel is partially filled with an inviscid, incompressible, constant density fluid of mass m_f and density ρ. When the vessel is in motion the free surface of the fluid is given by $y=h(x,t)$, with mean depth h_0, where $\mathbf{x}=(x,y)$ is a coordinate system fixed to the moving vessel. The base of the vessel is at $y=0$. We also define $\hat{\mathbf{X}}=(\hat{X},\hat{Y})$ to be a planar fixed coordinate system with origin at the point at which the vessel pivots. The fluid mass $m_f=\int_0^L \rho h(x,t)\,dx$ is independent of time. This pivoting TLD setup is of interest to engineers, because it is a good mechanism for suppressing torsional vibrations on bridges caused by aerodynamic effects (Xue et al., 2000; Chen et al., 2008), which are a danger to high sided vehicles (Chen and Cai, 2004). These studies include experiments as well as linear and nonlinear simulations, where the shallow water model is assumed for the fluid motion.

This configuration is the simplest coupling between fluid and vessel motion that allows rotation. The TLD configuration is simpler but only allows for translation of the vessel. This pendulum-slosh model was one of the first coupled models to be studied (Moiseev, 1953; Abramson et al., 1961; Moiseyev and Rumyantsev, 1968). Indeed, the linear equations of motion were first derived in Moiseyev and Rumyantsev (1968) by considering the added mass coefficients for the fluid and a Lagrangian construction for the two-dimensional vessel equation. They then went on to derive the characteristic relation for linear perturbations in terms of a general vessel geometry. In this paper we present three new results for this linear coupled problem: firstly, we give a new independent derivation of the governing equations and characteristic equation for the natural frequencies, confirming the result in Moiseyev and Rumyantsev (1968); secondly, we have discovered a new instability of this coupled system; and thirdly, we have discovered a 1:1 resonance in the system.

The instability range is surprising because it always occurs with the pivot point above the centre of mass of the quiescent fluid. If the fluid was a rigid body of length L and height h_0 and the pivot point was on the vertical centreline, the configuration would be unstable if and only if the pivot point was below the centroid:

$$l<\frac{1}{2}h_0.$$

By replacing the rigid body with a fluid in the interior, the instability can arise with the pivot point above the point $y=\frac{1}{2}h_0$. We have discovered the remarkable and exact formula:

$$(1+R)l<\frac{1}{2}h_0+\frac{1}{12}\frac{L^2}{h_0}, \tag{1.1}$$

for the instability threshold, where

$$R=\frac{m_v}{m_f}, \tag{1.2}$$

and m_v is the mass of the dry vessel. For example the instability threshold can be even greater than h_0 (above the still water level), depending on the values of R, h_0 and L. The depth ratio h_0/L of the fluid plays a key role. The effect of this instability on mechanical structures, such as the TLD for torsional bridge oscillations, would be catastrophic, leading to large unstable oscillations, which could ultimately cause structural damage.

This configuration is to contrasted with Cooker's experiment (Cooker, 1994), which is also pendular, but with two suspension points so the base of the vessel always remains horizontal. In this configuration the trivial solution is never unstable. It has however many other features of interest. It has been studied experimentally by Cooker (1994) and