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# On the dynamics of the fluid balancer

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ABSTRACT

This paper is concerned with the dynamics of a so-called fluid balancer; a hula hoop ring-like structure containing a small amount of liquid which, during rotation, is spun out to form a thin liquid layer on the outermost inner surface of the ring. The liquid is able to counteract unbalanced mass in an elastically mounted rotor. The paper derives the equations of motion for the coupled fluid–structure system, with the fluid equations based on shallow water theory. An approximate analytical solution is obtained via the method of multiple scales. For a rotor with an unbalance mass, and without fluid, it is well known that the unbalance mass is in the direction of the rotor deflection at sub-critical rotation speeds, and opposite to the direction of the rotor deflection at super-critical rotation speeds (when seen from a rotating coordinate system, attached to the rotor). The perturbation analysis of the problem involving fluid shows that the mass center of the fluid layer is in the direction of the rotor deflection for *any* rotation speed. In this way a surface wave on the fluid layer can counterbalance an unbalanced mass.



## 1. Introduction

A fluid balancer is used in rotating machinery to eliminate the undesirable effects of unbalanced mass. It has become a standard feature in (high-grade) household washing machines, and is also used in heavy industrial rotating machinery. Taking the washing machine fluid balancer as example, it consists of a hollow ring, like a hula hoop ring but typically with rectangular cross sections, which contains a small amount of liquid (typically brine, which has a relatively large density). The ring is most often attached on top of the drum. When it rotates at a high angular velocity $\Omega$ the liquid will form a thin liquid layer on the inner surface of the outermost wall, as sketched in Fig. 1.

Consider the situation where an unbalanced mass $m$ is present, for example due to the non-uniform distribution of clothes in a washing machine. The rotor has a critical angular velocity $\Omega_{cr}$ where the centrifugal forces are in balance with the forces due to the restoring springs. Below this velocity ($\Omega < \Omega_{cr}$) the mass center of the fluid will be located 'on the same side' as the unbalanced mass, as shown in the left part of Fig. 1. [Here $M$ indicates the mass of the empty rotor and $\mathcal{M}$ the mass of the contained liquid.] At a certain super-critical angular velocity $\Omega > \Omega_{cr}$ (say, during the spin drying process) the mass center of the liquid will move to the 'opposite side' of the unbalanced mass, as shown in the right part of Fig. 1, resulting in 'mass balance' and thus in reduced centrifugal forces and reduced oscillation amplitude of the rotor.

This is the working principle of the fluid balancer. The main idea appeared already in 1912, and US patent was granted in 1916 (Leblanc, 1916). The original layout consisted of one or several very narrow concentric channels (narrow in the radial

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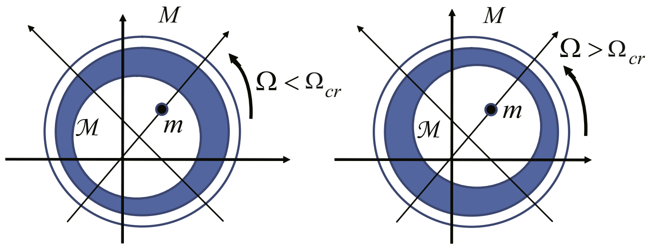


**Fig. 1.** Working principle of the fluid balancer.

direction but wide in the axial direction, i.e., perpendicular to the paper in Fig. 1) partially filled with, “liquid, or very small steel balls or metal fillings”. Leblanc's fluid balancer was discussed and criticized by Thearle (1932); and later also by Den Hartog (1985), in connection with a discussion of Thearle's balancing head of Thearle (1932). It is argued there that Leblanc's balancer cannot work with a liquid, only with steel balls, and thus that the invention was flawed. It appears that this is due to the very narrow channels which basically prevent the formation of surface waves. [Apparently it was Leblanc's idea that the fluid (mercury) layer would ‘break up’ and work just like steel balls.]

None the less, a complete automatic washing machine equipped with a fluid balancer was presented in 1940, and patented in 1945 (Dyer, 1945). The layout of the fluid balancer was very similar to the modern layouts, with a wide concentric channel, wide enough to allow for surface waves with large amplitudes.

The idea is thus not new; but recently there has been a renewed interest, both in industry and in academia. [There has also been a renewed interest in the so-called automatic dynamic balancer, as the balancer that uses steel balls running in a circular channel (or race) is called, van de Wouw et al., 2005; Green et al., 2006, 2008.]

Experimental fluid balancer studies have been carried out by Kasahara et al. (2000b) and Nakamura (2009). As to mathematical models, simple lumped mass models have been considered by Bae et al. (2002), Jung et al. (2008), Majewski (2010), Chen et al. (2011), and Urbiola-Soto and Lopez-Parra (2011). The first and the last two of these papers include experimental studies as well. The paper by Jung et al. (2008) includes a few numerical simulation results based on computational fluid dynamics.

It should be emphasized that the fundamental principle of operation of a ‘perfectly working’ fluid balancer can be understood in terms of the explanation of Thearle's balancing head, given in Den Hartog (1985, p. 37). But to get a fluid balancer to work perfectly is a delicate process and, thus, a more detailed understanding is desirable; in particular, a more detailed understanding of the fluid dynamics of the balancer. This is the main motivation behind the present work.

An important early paper dealing with *stable* motion (based on rather simple modeling of the fluid) is that of Ehrich (1967). The paper discusses both asynchronous and synchronous whirl. In the first case, the rotor is whirling with a frequency slightly different from the frequency of rotation; in the latter case, these two frequencies are exactly equal. An interesting conclusion is that, in the case of synchronous whirl of a rotor with just a small amount of trapped fluid (yet fully wetted cavity wall), the rotor behaves as if it were completely filled with fluid!

This suggests that the fluid balancer cannot work via synchronous whirl but rather must be based on the action of asynchronous whirl, at least if the cavity wall is fully wetted. The fact that the balancing wave tends to drift away from its ‘best’ position further supports the assumption of asynchronous whirl (Nakamura, 2009; Urbiola-Soto and Lopez-Parra, 2011).

More attention to fluid dynamic details has been given in many of the studies dealing with the *stability* of rotors partially filled with fluid/liquid; see e.g. Bolotin (1963) and Crandall (1995) for good overviews. A very recent review, covering also the fluid balancer, is given in Kaneko et al. (2014, Chapter 7). Most of these studies, such as those of Wolf (1968), Hendricks and Morton (1979), and Holm-Christensen and Träger (1991), are based on linear theory/linearization. While this is sufficient to determine the stability properties, it may be insufficient for modeling and understanding the dynamics of the fluid balancer. This is because one would expect that a finite-amplitude surface wave is necessary to counterbalance a finite unbalanced mass $m$.

Non-linear studies have been carried out by Jinnouchi et al. (1985), Berman et al. (1985), Colding-Jørgensen (1991), Kasahara et al. (2000a), and Yoshizumi (2011). Berman et al. (1985) focused on the case of asynchronous whirl and found, both by numerical analysis and by experiment, that non-linear surface waves can exist on the fluid layer in the form of hydraulic jumps, undular bores, and (what appears to be) solitary waves (or solitons). [An undular bore is a relatively weak hydraulic jump, with undulations behind it, Lighthill, 1978, p. 180. As to a solitary wave, described by the square of a hyperbolic secant function, sech, it should be noted that such a solution/wave exists only in a doubly infinite (i.e. non-periodic) domain. In the *periodic* domain of the rotor vessel, the solution which corresponds to a solitary wave is described by the square of a Jacobian elliptic cosine function, cn, and is termed a cnoidal wave, Lighthill, 1978; Whitham, 1999.] Jinnouchi et al. (1985) found a hydraulic jump solution numerically, independently from Berman et al. (1985). Colding-Jørgensen (1991) concentrated on the hydraulic jump solution, following the analytical solution approach given in Berman et al. (1985). Contrary to this approach, the studies of Kasahara et al. (2000a) and Yoshizumi (2011) are purely numerical.