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Modeling and analysis of water-hammer in coaxial pipes

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ABSTRACT

The fluid-structure interaction is studied for a system composed of two coaxial pipes in an annular geometry, for both homogeneous isotropic metal pipes and fiber-reinforced (anisotropic) pipes. Multiple waves, traveling at different speeds and amplitudes, result when a projectile impacts on the water fillinhe annular space between the pipes. In the case of carbon fiber-reinforced plastic thin pipes we compute the wavespeeds, the fluid pressure and mechanical strains as functions of the fiber winding angle. This generalizes the single-pipe analysis of J. H. You, and K. Inaba, *Fluid-structure interaction in water-filled pipes of anisotropic composite materials*, J. Fl. Str. 36 (2013). Comparison with a set of experimental measurements seems to validate our models and predictions.

1. Introduction

This article is part of a series of papers (Ho You and Inaba, 2013; Bitter and Shepherd, 2013, 2013; Inaba and Shepherd, 2008; Perotti et al., 2013) devoted to the investigation of water-hammer problems in fluid-filled pipes, both from the experimental and theoretical perspective. Water-hammer experiments are a prototype model for many situations in industrial and military applications (e.g., trans-ocean pipelines and communication networks) where we have fluid-structure interaction and a consequent propagation of shock-waves. After the pioneering work of Korteweg (1878) and Joukowsky (1900), who modeled water-hammer waves by neglecting inertia and bending stiffness of the pipe, a more comprehensive investigation, developed by Skalak (1956) in the Fifties, considered inertial effects both in the pipe and the fluid, including longitudinal and bending stresses of the pipe. Skalak combined the Shell Theory for the tube deformation and an acoustic model of the fluid motion. He shows there is a coexistence of two waves traveling at different speeds: the precursor wave (of small amplitude and of speed close the sound speed of the pipe wall) and the primary wave (of larger amplitude and lower speed). Additionally, a simplified four-equation one-dimensional model is derived based on the assumption that pressure and axial velocity of the fluid are constant across cross-sections (Skalak, 1956). Later studies of Tijsseling (1993), Tijsseling (2003) and Tijsseling (2007) have regarded modeling of isotropic thin pipes including an analysis of the effect of thickness on isotropic pipes based on the four-equation model (Tijsseling, 2007). While all these papers consider the case of elastically isotropic pipes, the investigation of anisotropy in water-filled pipes of composite materials was first obtained in Ho You and Inaba (2013) where stress wave propagation is investigated for a system composed of water-filled thin pipe with symmetric winding angles $\pm\theta$. In the same geometry, a platform of numerical computations, based on the finite element method, was developed in Perotti et al. (2013) to describe the fluid-structure interaction during

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shock-wave loading of a water-filled carbon-reinforced plastic (CFRP) tube coupled with a solid-shell and a fluid solver. More complex situations involve systems of pipes mounted coaxially where the annular regions between the pipes can be filled with fluid. In this scenario, Bürmann has considered the modeling of non-stationary flow of compressible fluids in pipelines with several flow sections (Bürmann, 1975). His approach consists of reducing the system of partial differential equations governing the fluid-structure interaction in coaxial pipes into a 1-dimensional problem by the Method of Characteristics. Later works have appeared on the modeling of sound dispersion in a cylindrical viscous layer bounded by two elastic thin-walled shells (Levitsky et al., 2004) and of the wave propagation in coaxial pipes filled with either fluid or a viscoelastic solid (Cirovic et al., 2002).

Motivated by the recent experimental effort of J. Shepherd's group on the investigation of the water-hammer in annular geometries (Beltman et al., 1999; Beltman and Shepherd, 2002; Bitter and Shepherd, 2013, 2013), we extend the modeling work of Tijsseling (2007) and Ho You and Inaba (2013) to investigate the propagation of stress waves inside an annular geometry delimited by two water-filled coaxial pipes, in elastically isotropic and CFRP pipes. A projectile impact causes propagation of a water pressure wave causing the deformation of the pipes. Positive extension in the radial direction of the outer pipe, accompanied by negative extension (contraction) in the radial direction of the internal pipe, causes an increase in the annular area thus activating the fluid-structure interaction mechanism.

The architecture of the paper is as follows. After reviewing the work of You and Inaba on the modeling of elastically anisotropic pipes, we present the six-equation one-dimensional model (Paragraph 2.2) that rules the fluid-solid interaction in a two-pipe system. In Section 3 we compare our theoretical findings with experimental data obtained during a series of water-hammer experiments. Finally, in the case of fiber reinforced pipes, the wave propagation and the computation of hoop and axial strain are described in full detail in Paragraph 3.3.

2. Thin pipes modeling

2.1. One-dimensional fluid-structure modeling

According to the technique of Tijsseling (2007), one-dimensional governing equations for the liquid and the pipes can be obtained upon averaging out the standard balance laws in the radial direction. By adopting a cylindrical coordinates system, this approach is based upon the assumption that the behavior of water velocity and pressure depend only on the spatial variable z. In what follows we define one-dimensional cross-averaged quantities and obtain the corresponding field equations.

2.1.1. Governing equations for the fluid

The balance laws in the coordinate system (r,z) for the fluid read (Tijsseling, 1993)

(2-d) Axial motion equation:

$$\rho_w \frac{\partial v_z}{\partial t} + \frac{\partial p}{\partial z} = 0,$$

(2-d) Radial motion equation:

$$\rho_w \frac{\partial v_r}{\partial t} + \frac{\partial p}{\partial r} = 0,$$

(2-d) Continuity equation:

$$\frac{1}{K}\frac{\partial p}{\partial t} + \frac{\partial v_z}{\partial z} + \frac{1}{r}\frac{\partial (r v_r)}{\partial r} = 0.$$

Here $v_z(r,z,t)$ and $v_r(r,z,t)$ are, respectively, the axial and radial velocity of the fluid and $p(r,z,t)$ is the pressure; K is the bulk modulus of the fluid and ρ_w is the density of the fluid. We now introduce the cross-sectional averaged (one-dimensional) velocity and pressure, defined respectively as

$$V(z,t) := \frac{1}{\pi\left(R_2^2 - (R_1+e_1)^2\right)} \int_{R_1+e_1}^{R_2} 2\pi r\, v_z(r,z,t)\, dr, \tag{2.1}$$

$$P(z,t) := \frac{1}{\pi\left(R_2^2 - (R_1+e_1)^2\right)} \int_{R_1+e_1}^{R_2} 2\pi r\, p(r,z,t)\, dr. \tag{2.2}$$

We are in a position to introduce the one-dimensional equations of balance for the fluid, which are