ARTICLE IN PRESS

Journal of Fluids and Structures ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

A robust high-resolution method for the time–frequency analysis of vortex-induced-vibration signals

Sau-Lon James Hu [a,*], Wenlong Yang [b,c], Huajun Li [d]

[a] *Department of Ocean Engineering, University of Rhode Island, Narragansett, RI 02882-1197, USA*
[b] *China International Marine Containers (Group) LTD., Shenzhen 518067, China*
[c] *CIMC Offshore Engineering Institute Company Limited, Yantai 264670, China*
[d] *Shandong Provincial Key Lab of Ocean Engineering, Ocean University of China, Qingdao 266100, China*

ARTICLE INFO

Article history:
Received 20 January 2014
Accepted 5 August 2014

Keywords:
Vortex-induced vibration
Time–frequency analysis
Short Time Prony Transform
State-space model

ABSTRACT

The dynamic interaction between ocean current and marine riser is complex in nature, and the riser's vortex-induced vibration (VIV) due to the current often strongly exhibits a non-stationary phenomenon. For investigating the time-varying dominant frequencies of the VIV motion, a joint time–frequency analysis is necessary. Traditional methods for time–frequency analysis include the *Short Time Fourier Transform* (STFT) and *Wavelet Transform* (WT) methods, though both methods have significant drawbacks. Specifically, the STFT method suffers frequency resolution and leakage problems, while the WT method is highly sensitive to its basic wavelet selection. This paper newly introduces a robust high-resolution method, named the STPT-SS method, which is the *Short Time Prony Transform* (STPT) using a *State-Space* (SS) model. In particular, the STPT algorithm contributes to the high-resolution feature of the proposed method, and the SS model to the robustness. Using test VIV data that include a synthesized signal and measurements from laboratory and field experiments, the STPT-SS method is found to significantly outperform the STFT and WT methods in the time–frequency analysis.

1. Introduction

Flow- or vortex-induced vibrations occur in many mechanical and civil engineering systems, such as bridges, high-rise buildings, and offshore structures. One particular instance which is of vital interest to offshore engineering is the vortex induced vibrations (VIV) of marine risers, or long slender cylinders, due to ocean currents (Sarpkaya, 2010; Wu et al., 2012). As the dynamic interaction between marine riser and ocean current is complex in nature, the riser's VIV motion often strongly exhibits a non-stationary phenomenon.

Knowing the riser's VIV motion is crucial to the estimation of the vortex-induced dynamic stresses on the riser, which in turn allows one to predict the fatigue damage of the riser. Many experimental investigations on flexible slender cylinders have been carried out to better understand the dynamic characteristics of the VIV motion (Hamdan et al., 1996; Kaasen and Lie, 2003; Trim et al., 2005; Vandiver et al., 2009; Song et al., 2011; Ikoma et al., 2012). To identify the dominating frequencies and frequency ranges of the VIV motion, many researchers have chosen to employ the Discrete Fourier transform (DFT), which is implemented by the Fast Fourier transform (FFT). Although running FFT is simple and quick, there are drawbacks of using it, such as frequency resolution and leakage. Furthermore, conducting a pure frequency analysis does not provide information about the particular time at which the

---

[*] Corresponding author. Tel.: +1 401 874 6688; fax: +1 401 874 6837.
E-mail addresses: hu@egr.uri.edu (S.-L. James Hu), oucwillow@gmail.com (W. Yang), huajun@ouc.edu.cn (H. Li).

http://dx.doi.org/10.1016/j.jfluidstructs.2014.08.013
0889-9746/© 2014 Elsevier Ltd. All rights reserved.

Please cite this article as: James Hu, S.-L., et al., A robust high-resolution method for the time–frequency analysis of vortex-induced-vibration signals. Journal of Fluids and Structures (2014), http://dx.doi.org/10.1016/j.jfluidstructs.2014.08.013

dominant frequencies occur or their corresponding durations. In order to retain time information, windowed DFT, also known as Short Time Fourier Transform (STFT) has been applied. Unfortunately, the frequency resolution will suffer even further while implementing STFT, as the frequency resolution in DFT is inversely proportional to the time window size.

Another common time–frequency analysis is the Wavelet Transform (WT) technique, which uses wavelets to decompose any signal for detailed analysis with multiple time–frequency resolution. In this method, an appropriate wavelet is compared with the signal over a defined time period and a coefficient is obtained that is basically a correlation of a signal with the wavelet. The resulting WT is a scalogram which is expressed in terms of scales or frequency bands, instead of harmonic frequencies. During the last two decades, a number of articles have conducted the time–frequency analysis of the VIV signals using the WT technique (Hamdan et al., 1996; Jubran et al., 1998; Kaasen and Lie, 2003; Mukundan et al., 2010; Larsen et al., 2012; Ikoma et al., 2012). While it is easy to implement WT if a basic (mother) wavelet has been properly chosen, the mother wavelet selection may be rather arbitrary and can significantly affect the resulting scalograms. As concluded in previous studies (Hamdan et al., 1996; Jubran et al., 1998), the selection of the most suitable type of wavelet, as well as its parameters, still represents a major challenge for the time–frequency analysis of the VIV motion.

Although much effort has been done for the frequency and time–frequency analysis of the complicated fluid structure interaction phenomenon, an effective method for analyzing the VIV motion is still lacking. The purpose of this paper is to newly introduce a time–frequency analysis method, which is robust and of high-resolution, on analyzing the time-varying frequency components of VIV motion signals. The proposed time–frequency analysis method is named the *STPT-SS* method, which is the *Short Time Prony Transform* (STPT) using a *State-Space* (SS) model. While the STPT algorithm contributes to the high-resolution time–frequency analysis, the robustness of the method comes from using an SS model (Hu et al., 2013). The notion of "short time" of the STPT-SS method is similar to that of STFT—using a rectangular window sliding through the whole signal. In the STPT-SS method, the signal decomposition of each windowed segment is based on a Prony series, which is the sum of a finite number of complex exponential components (de Prony, 1795; Marple, 1987). In contrast to Fourier-based techniques which assign the frequency of each component directly, Prony-based techniques involve a step of estimating the frequency of each component before computing its corresponding amplitude. Without the periodic assumption for the signal to be analyzed, Prony-based techniques do not suffer the frequency resolution and leakage problems associated with Fourier-based techniques. Another advantage of Prony-based method is that it requires only a short duration segment to gain high frequency resolution in the decomposition process (Hu et al., 2013). This advantage would be particularly attractive for the time–frequency signal analysis. While the frequency resolution of the STFT, which is limited by the duration of the associated sliding window, is often an issue for the time–frequency signal analysis, the Prony-based method would overcome the frequency resolution issue and thus offer an excellent building block for the time–frequency signal analysis. Since the technique to decompose each segment into Prony series is based on a state-space model, it is termed the *Prony-SS method* to distinguish it from the traditional *Prony's method*. When comparing the Prony-SS method and the traditional Prony's method, the former possesses better numerical conditioning and stability (Hu et al., 2013). While the traditional Prony's method is numerically sensitive to the noise embedded in the signal, the sampling rate and even the computer round-off error, the proposed Prony-SS method is very robust to the sampling rate and round-off error, and has a built-in noise rejection mechanism via the usage of truncated singular value decomposition (TSVD).

In the numerical study, three kinds of VIV data sets are employed to test and evaluate the proposed method: (i) synthesized benchmark signals proposed by Mukundan (2008), (ii) experimental data from Norwegian Deepwater Program (NDP) (Trim et al., 2005), and (iii) field study data from Gulf Stream Test (GST) (Vandiver et al., 2009). The synthesized signals contain multiple closely spaced frequencies, but stationary in nature, and will be employed to demonstrate the performance of the Prony-SS method. In the time–frequency analysis, the data utilized in this study are NDP acceleration measurements at both in-line and cross-flow directions, and GST strain measurements at two quadrants orthogonal to each other, where data measured at different axial locations of risers will be investigated.

2. Preliminaries

The first part of this section reviews the mathematical model for the dynamics of a marine riser. It describes that the riser's response is nonlinear and non-stationary, but can be reasonably treated to be piecewise linear and stationary within a short duration. The second part briefly reviews two traditional methods for the time–frequency analysis of a non-stationary signal: Short Time Fourier Transform (STFT) and Wavelet Transform (WT) methods. Those two methods are to be tested against a newly proposed Prony-based method.

2.1. Dynamic model of marine riser

The dynamic response of a marine riser can generally be modeled by a second-order differential equation (Hoen and Moe, 1999)

$$\mathbf{M}_s\ddot{\mathbf{q}}(t)+\boldsymbol{\zeta}_s\dot{\mathbf{q}}(t)+\mathbf{K}_s\mathbf{q}(t)=\mathbf{f}(\ddot{\mathbf{q}},\dot{\mathbf{q}},\mathbf{q},t), \tag{1}$$

where $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ are vectors of generalized acceleration, velocity and displacement, respectively; $\mathbf{f}(\ddot{\mathbf{q}},\dot{\mathbf{q}},\mathbf{q},t)$ is the forcing function; and $\mathbf{M}_s$, $\boldsymbol{\zeta}_s$ and $\mathbf{K}_s$ are the mass, damping and stiffness matrices of the riser structure, respectively. The forcing function $\mathbf{f}(\ddot{\mathbf{q}},\dot{\mathbf{q}},\mathbf{q},t)$ can be decomposed into a sum of elements being proportional to the acceleration, velocity and

Please cite this article as: James Hu, S.-L., et al., A robust high-resolution method for the time–frequency analysis of vortex-induced-vibration signals. Journal of Fluids and Structures (2014), http://dx.doi.org/10.1016/j.jfluidstructs.2014.08.013