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Mathematical models for fluid–solid interaction and their numerical solutions

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ABSTRACT

This paper considers various approaches used currently for the fluid–solid interaction problem and associated computational methodologies. The validity of the mathematical models for fluid–solid interaction is established based on the consistency in the use of continuum mechanics principles and whether the interaction between the solid and the fluid is inherent in the mathematical model or is established external to the mathematical model through interface constraint equations. Computational methodologies are considered from the point of view of unconditional stability, accuracy, and adaptivity of the numerical schemes employed. In particular, the paper establishes that fluid–solid interaction physics must be intrinsic in the mathematical model(s), the mathematical models for fluid and solid must have the same description, either Eulerian with transport, Lagrangian, or Eulerian without transport. Since fluids require the Eulerian description with transport, a similar description for solid matter (hypo-elastic solid) indeed provides a mathematical model for fluid–solid interaction in which the fluid–solid interaction is intrinsic in the mathematical model. The mathematical models for solid matter in the Lagrangian description or in the Eulerian description without transport and for fluids in the Eulerian description with transport can never interact due to fundamental differences in their derivations and the basic assumptions employed. For example, the Eulerian description with transport for fluids precludes material point displacements, which are intrinsically present in the Lagrangian description and the Eulerian description without transport, and they are needed for interaction of the fluid with the solid. The mathematical models for solid matter in the Lagrangian description, the Eulerian description without transport, and for fluids in the Eulerian description with transport are presented to illustrate why fluid–solid interaction is not possible with these mathematical models. The ALE methodologies using the mathematical models in Lagrangian and Eulerian descriptions have been carefully evaluated and are demonstrated to be invalid for a consistent formulation of a fluid–solid interaction problem. Some numerical studies for simple model problems are presented to demonstrate various issues discussed here. The present study establishes the possible mathematical models and their limitations within the current knowledge of continuum mechanics that provide correct model for fluid–solid interaction.

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1. Introduction, literature review and scope of work

1.1. Introduction

Mathematical modeling and numerical simulations of fluid–solid interaction have long been subject of interest. The most challenging and interesting class of applications are those in which the fluid–solid interface experiences finite motion and/or deformation. If one uses a mathematical modeling approach based strictly on the principles of continuum mechanics, then the derivation(s) of the mathematical model(s) for solid matter using Lagrangian description with appropriate measures of finite strain and stress are most natural and useful. In this description the locations $\boldsymbol{x}$ or $\overline{\boldsymbol{x}}$ are indeed locations of the material point in the reference and current configurations and $\overline{\boldsymbol{x}}=\boldsymbol{x}+\boldsymbol{u}(\boldsymbol{x},t)$ holds. The displacements of material point $\boldsymbol{u}(\boldsymbol{x},t)$ are monitored during the entire evolution. The physics of deformation for solid matter necessitates this, so that kinematic relations and the associated constitutive theories can be derived. Thus, for solid matter we follow each material point during evolution through its displacement. In the case of fluids, the physics of motion and deformation is quite different than in solid matter.

Consider a simple but common occurrence of flow past an impermeable boundary. Fluid particles attached to the impermeable boundary are immobile, whereas the location next to them in the fluid domain are occupied by different fluid particles during the evolution. That is, as the evolution proceeds, different fluid particles rub against these stationary fluid particles attached to the impermeable boundary and move on. In the flow domain away from the impermeable boundaries similar phenomenon exists but the particles rubbing against each other may also be experiencing rigid body motion, that is, they may have their own velocities. As well known, this phenomenon of fluid particles rubbing against each other collectively leads to conversion of mechanical energy into entropy production, which is the basis of Newton's law of viscosity. Thus in fluids: (a) motion of the material points may be so complex that a Lagrangian description may not be possible to describe it and (b) the strains are relatively small and do not play a significant role in the constitutive theory, instead velocity gradients are largely responsible for the stress field. We note that complex motion of fluid particles is difficult to monitor, but fortunately it is not required in the mathematical description of fluids as the strains are negligible.

This short description of the physics of the fluid motion suggests that we can entertain an alternate approach for deriving the mathematical model for fluid motion in which we consider fixed location in the flow domain and monitor the state of the fluid at these locations during evolution. In this approach, the fixed spatial locations are occupied by different fluid particles during the evolution. Thus, these fixed locations are indeed current positions of some fluid particles. Hence it is appropriate to designate them by $\overline{\boldsymbol{x}}$. At a fixed location we can monitor velocities, temperature, stresses, etc. but not the displacements (and they are not needed in the kinematic description of fluid motion). At a fixed location $\overline{\boldsymbol{x}}$ we do not know which fluid particle it is at time t, nor do we know where it is going or came from. This, of course, is all due to the fact that we gave up monitoring displacements of the material points. In deriving the details of the mathematical model for such physics we must consider $\overline{x}_i$ locations and material particle transport as the particles arriving at the location $\overline{\boldsymbol{x}}$ are due to the transport. In this paper we refer to such description as the Eulerian description (due to $\overline{\boldsymbol{x}}$) with transport. We remark that this is quite different than the Eulerian description of solids in which $\overline{\boldsymbol{x}}=\boldsymbol{x}+\boldsymbol{u}$ holds and thus the mathematical models can either be derived using $(\boldsymbol{x},t)$, the Lagrangian description or $(\overline{\boldsymbol{x}},t)$, the Eulerian description. The two descriptions are equivalent due to uniqueness of displacements $\boldsymbol{u}$. In fluids we are obviously stuck with fixed location $\overline{\boldsymbol{x}}$, the current position of some material particle and material transport but without knowledge of displacements $\boldsymbol{u}$. There is no alternative to this description based on the currently used principles and concepts of continuum mechanics for deriving mathematical model of fluid motion.

From the point of view of this paper, an important question is: do these two approaches of deriving the mathematical model for fluid and solid provide the ability of interaction between them? We provide an answer to this question in this paper.

1.2. Literature review

A large majority of the recently published literature and on going research work in the area of fluid–solid interaction uses the Arbitrary Lagrangian Eulerian (ALE) approach for mathematical models as a starting point for designing various computational strategies for numerical calculations of the fluid–solid interaction evolution. The ALE concept was first published by Hirt et al. (1974) based on the work of Noh (1964) and subsequently by Donea et al. (1982), Liu and Chang (1985) and Liu et al. (1986). In this approach the fluid and solid descriptions are Lagrangian and Eulerian. Based on Donea et al. (1982), "ALE has no basic dependence on particles and treats the computational mesh as a reference frame which may be moving with arbitrary velocity $\tilde{v}$." It is claimed that, based on the choice of $\tilde{v}$, Lagrangian and Eulerian descriptions are recoverable from a single mathematical model, which is subsequently utilized for numerical computations. Hughes et al. (1981) used ALE approach for modeling fluid sub-domains of many fluid–solid interaction and free surface problems. Tezduyar et al. (1992a,b) presented DSD/SST (deforming-spatial-domain/stabilized space–time) procedure developed in Tezduyar et al. (1992b) for finite element computations involving moving boundaries and interfaces. Mesh moving techniques in fluid–solid interactions using DSD/SST are presented by Tezduyar and Benney (2003). In chapter 14 of *The Encyclopedia of Computational Mechanics*, Donea et al. (2004) present details of ALE published in their previous papers

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