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A fast harmonic balance technique for periodic oscillations of an aeroelastic airfoil

Honghua Dai, Xiaokui Yue *, Jianping Yuan, Dan Xie

College of Astronautics, Northwestern Polytechnical University, Xi'an 710072, PR China

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ABSTRACT

The harmonic balance (HB) method is utilized to obtain the periodic solutions for the two-dimensional airfoil with cubic nonlinearity in pitch undergoing subsonic flow. In the course of formulating the HB algebraic system, the manipulation software Mathematica is employed to deal with the complex Fourier coefficients involved with the nonlinear term. In general, to solve the HB algebraic system, either a symbolic calculation or a numerical approximation of the Jacobian matrix is required in each iteration, which is computationally expensive. To remedy this drawback, the Jacobian matrix is explicitly derived in this paper. The effects of exploiting the explicit Jacobian matrix on the accuracy and efficiency of the HB method are investigated, through comparing with the case using a numerical Jacobian matrix calculated by a three-point difference technique. Moreover, the spectral analysis is applied to the periodic motions by the numerical method to provide insight into the distribution of the dominant frequencies, so as to provide a reasonable suggestion for the truncation of the Fourier series expansion in the HB method. In addition, a frequency modulation phenomenon is identified in the pitch motions via spectral analysis, whose effect on the accuracy of the HB method is examined both numerically and analytically. Finally, illustrative examples validate that the HB method with as many harmonics as the spectral analysis suggests can yield sufficiently accurate solutions.

1. Introduction

Aeroelasticity is considered to be a source of instability and vibration problems for aircraft wings; it is concerned with the physical phenomena which involve significant mutual interaction among inertial, elastic and aerodynamic forces (Dowell et al., 1995). The nonlinearities mostly encountered in aeroelastic system are classified as structural and aerodynamics ones (Patil and Hodges, 2004). The former mainly refers to nonlinear stiffness resulting from large amplitude vibration, freeplay and/or hysteresis nonlinearities. One prominent consequence of the structural nonlinearity is that, once the flutter speed is exceeded, the wing goes into a bounded oscillation rather than an exponentially increasing oscillation predicted by the linear aeroelastic model. A vast of studies have been conducted to investigate the two DOF wing model with cubic (Woolston et al., 1955; Price et al., 1995; Lee et al., 1997; Liu et al., 2000), freeplay (Hauenstein and Laurenson, 1989; Alighanbari and Price, 1996; Chung et al., 2007; Li et al., 2012) and hysteresis (Liu et al., 2012; Chung et al., 2009) nonlinearities. The two DOF airfoil model with cubic nonlinearity in the pitch motion is our present concern. Experimental studies of the two DOF airfoil model have been conducted by a number of investigators

* Corresponding author.
E-mail addresses: daihonghua@gmail.com (H. Dai), xkyue@nwpu.edu.cn (X. Yue), jyuan@nwpu.edu.cn (J. Yuan), xiedan2151@163.com (D. Xie).

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Nomenclature

a_h	non-dimensional distance from airfoil mid-chord to elastic axis
b	airfoil semi-chord
C_h	damping coefficient in plunge
$C_L(\tau)$	coefficient of linear aerodynamic force
$C_M(\tau)$	coefficient of linear aerodynamic moment
C_α	torsion damping coefficient
DOF	degree of freedom
FFT	fast Fourier transformation
$G(\xi)$	structural nonlinearity in plunge
h	plunge deflection
HBn	harmonic balance method with n harmonics
I_α	wing mass moment of inertia about elastic axis
LCO	limit cycle oscillation
m	airfoil mass
$M(\alpha)$	structural nonlinearity in pitch
r_α	radius of gyration about elastic axis
S	airfoil static moment about elastic axis
t	time
U	free-stream velocity
U^*	$U/b\omega_\alpha$, non-dimensional velocity
U_L^*	non-dimensional linear flutter speed
x_α	non-dimensional distance from airfoil elastic axis to center of mass
α	pitch angle
β, γ	coefficients for cubic springs in pitch and plunge
ϵ_1, ϵ_2	constants in Wagner's function
ϕ	Wagner's function
μ	$m/\pi\rho b^2$
ρ	air density
τ	Ut/b non-dimensional time
ω	fundamental circular frequency of the motion
$\omega_\xi, \omega_\alpha$	natural frequencies in plunge and pitch
$\overline{\omega}$	ω_ξ/ω_α, frequency ratio
ξ	h/b, non-dimensional plunge deflection
ψ_1, ψ_2	constants in Wagner's function
ζ_α, ζ_ξ	viscous damping ratios in pitch and plunge

(Yang and Zhao, 1988; Hauenstein and Laurenson, 1989; Dietz et al., 2006; Abdelkefi et al., 2013). The experimental results can offer a standard to the theoretical studies to compare with. However, some inherent limitations exist in the experimental studies, such as big economic cost and sensitivity to the experimental setup (Dowell, 1975).

For theoretical studies, exact analytical solutions are rarely available due to the existence of nonlinearities. To the authors' knowledge, there exist four numerical schemes for the present airfoil problem. First, the finite difference method (FDM) (Price et al., 1994) is the most straightforward one, which directly discretizes the original governing integro-differential equations into large-scale ordinary differential equations (ODEs). Second, a set of four integral transformations was proposed by Lee et al. (1997) to deal with the integral terms in the original system, which has been adopted in most of the recent numerical investigations (Lee et al., 1999; Liu and Dowell, 2004; Liu et al., 2007, 2012). Third, Alighanbari and Price (1996) dealt with the integral terms through a twice differentiation technique. Fourth, Abdelkefi et al. (2013) transformed the integro-differential equations into a system of six pure differential equations based on the Sears and Pade approximations. For all the four schemes, the resulting ODEs can be integrated by the numerical integration method directly.

Prior to the study of Lee et al. (1997), semi-analytical methods (Yang and Zhao, 1988; Price et al., 1995) were only developed for the airfoil model with quasi-steady aerodynamics, wherein no integral terms are involved. The semi-analytical methods have been intensely developed for solving the system of eight coupled ODEs derived in Lee et al. (1997). These include the HB method (Lee et al., 2005), the perturbation incremental method (Chung et al., 2007), the high-dimensional harmonic balance method (Liu et al., 2007), and the incremental harmonic balance method (Liu et al., 2012). The HB method dates to the Galerkin method as early as in 1915. The Galerkin method is a powerful tool in the computational mechanics community for finding the approximate solutions of mechanical systems modeled by the nonlinear ODEs, which eliminates the single (spatial/temporal) coordinate to generate a system of nonlinear algebraic equations (NAEs). As for the systems modeled by partial differential equations (PDEs) in both space and time coordinates, the Galerkin method can also be used to discretize the spacial coordinates to result in a system of coupled ODEs in time; see Dowell (1966) and Tseng and Dugundji (1970). The Galerkin method working in the time domain becomes the HB method, which is probably the most frequently used method to deal with strongly nonlinear dynamical problems (Stoker, 1950; Hayashi, 1964; Liu and Dowell, 2004; Dai et al., 2013). It assumes a Fourier series expansion for the desired periodic solution and then obtains the NAEs of the coefficients through balancing each harmonic. If only one harmonic is considered, the HB method reduces to the equivalent linearization method or otherwise known as the describing function method. This method is well suited for obtaining periodic solutions of the dynamical systems in nature, relying on the properties of the assumed trigonometric trial functions. The HB method would be efficient enough, if the desired solution for a dynamical system can be represented by a small number of dominant harmonics. However, when the response of the system is complex, a large number of harmonics must be included to provide a sufficiently accurate solution, which would increase the computational cost dramatically. Liu and Dowell (2004) employed the HB method with up to nine harmonics to successfully detect the secondary Hopf bifurcation of the two DOF airfoil; nevertheless, large numbers of symbolic derivations are inevitable.

One objective of the present study is to reduce the large amount of computational costs arising from the application of the traditional high order HB method (Liu and Dowell, 2004). Firstly, to derive the complex Fourier coefficients in the HB algebraic system, the Mathematica is employed. Secondly, the explicit Jacobian matrix is derived to reduce the