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State-space model identification and feedback control of unsteady aerodynamic forces

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ABSTRACT

Unsteady aerodynamic models are necessary to accurately simulate forces and develop feedback controllers for wings in agile motion; however, these models are often high dimensional or incompatible with modern control techniques. Recently, reduced-order unsteady aerodynamic models have been developed for a pitching and plunging airfoil by linearizing the discretized Navier–Stokes equation with lift-force output. In this work, we extend these reduced-order models to include multiple inputs (pitch, plunge, and surge) and explicit parameterization by the pitch-axis location, inspired by Theodorsen's model. Next, we investigate the naïve application of system identification techniques to input–output data and the resulting pitfalls, such as unstable or inaccurate models. Finally, robust feedback controllers are constructed based on these low-dimensional state-space models for simulations of a rigid flat plate at Reynolds number 100. Various controllers are implemented for models linearized at base angles of attack $\alpha_0 = 0^\circ$, $\alpha_0 = 10^\circ$, and $\alpha_0 = 20^\circ$. The resulting control laws are able to track an aggressive reference lift trajectory while attenuating sensor noise and compensating for strong nonlinearities.

1. Introduction

Time-varying fluid flows are ubiquitous in modern engineering and in the life sciences, and controlling the corresponding unsteady aerodynamic forces and moments poses both a challenge and an opportunity. Biological propulsion illustrates the potential utilization of unsteady forces for engineering design (Daniel, 1984; Allen and Smits, 2001; Clark and Smits, 2006; Dabiri, 2009). It is observed that birds, bats, insects, and fish routinely exploit unsteady fluid phenomena to improve their propulsive efficiency, maximize thrust and lift, and increase maneuverability (Birch and Dickinson, 2001; Combes and Daniel, 2001; Sane, 2003; Wang, 2005; Wu, 2011; Shelley and Zhang, 2011). They achieve this performance with robustness to external factors, such as gust disturbances and weather, rapid changes in flight conditions, and even gross bodily harm. At the same time, they do so with fixed actuators (wing muscles) and a limited number of noisy, distributed sensors throughout the body. As uninhabited aerial vehicles (UAVs) become smaller and lighter, robust unsteady aerodynamic control will become increasingly important during agile maneuvers and gust disturbances.

Many aerodynamic models used for flight control rely on the quasi-steady assumption that forces and moments depend in a static manner on parameters such as relative velocity and angle of attack. In essence, the assumption is that maneuvers

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Nomenclature

$(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C})$	state-space model for transient lift
$(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C})_r$	reduced-order model of order r
a	pitch axis with respect to 1/2-chord
b	curvature parameter for step-up maneuvers
c	chord length of plate
C_L	lift coefficient $[C_L \triangleq 2L/\rho U_\infty^2 c]$
C_α	lift coefficient slope in α
e	noisy error signal
f	frequency of maneuver
$G(s)$	transfer function for transient lift
G_a	actuator model
g	horizontal position of plate
$\mathcal{H}_i$	i-th Markov parameter
h	vertical position of plate
k	reduced frequency $[k \triangleq \pi f c/U_\infty]$
L	lift force
L_d	desired loop shape
$\mathcal{L}$	Laplace transform
M	amplitude of motion
n	sensor noise
Re	Reynolds number $[\mathrm{Re} \triangleq cU_\infty/\nu]$
r	reduced-order model order
r_L	reference lift
St	Strouhal number $[\mathrm{St} \triangleq fM/U_\infty]$
s	Laplace variable (dimensionless)
t	time (dimensional)
$\boldsymbol{U}$	vector of input motion
U_∞	free stream velocity
$\boldsymbol{u}$	input to state-space model
$\boldsymbol{x}$	state of state-space model
$\boldsymbol{Y}$	vector of measurements
$\boldsymbol{y}$	output of state-space model
α	angle of attack of plate
α_e	effective angle of attack
α_0	base angle of attack
Δt_c	coarse time-step
Δt_f	fine time-step
ν	kinematic viscosity
ρ	fluid density
τ	time (dimensionless) $[\tau \triangleq tU_\infty/c]$
τ_h	hold time
τ_r	ramp time

are sufficiently slow so that the flow has time to equilibrate. While these models work well for conventional aircraft, they do not describe the unsteady aerodynamic forces that are important for small, agile aircraft to avoid obstacles, respond to gusts, and track potentially elusive targets. Small, lightweight aircraft have a lower stall velocity. Therefore, gusts and rapid motions excite large reduced frequencies, $k = \pi f c/U_\infty$, and Strouhal numbers, $\mathrm{St} = fM/U_\infty$, where f and M are the frequency and the amplitude of motion, respectively. Henceforth, lengths are nondimensionalized by the chord length c, velocities by the free-stream velocity U and time by c/U_∞. Loosely speaking, large reduced frequencies are excited when wing motion is so fast that unsteady flow structures do not have time to convect an entire chord length before new structures are formed. Large Strouhal numbers are excited by wing motions that are a combination of fast and large amplitude, and these typically result in complex wake structures. The Strouhal number and reduced frequency may be varied independently by the choice of frequency f and amplitude M.

There exist a wide range of unsteady aerodynamic models in the literature (Dowell and Hall, 2001; Leishman, 2006; Amsallem et al., 2010). The classical unsteady models of Wagner (1925) and Theodorsen (1935) are still used extensively, and they provide a standard of comparison for the linear models that follow them (Bruno and Fransos, 2008). Wagner's model constructs the lift in response to arbitrary input motion by convolving the time derivative of the motion with the analytically computed step response, also called the *indicial response*. Linear indicial response models are a mainstay of the unsteady aerodynamics (Tobak, 1954) and aeroelasticity (Marzocca et al., 2002; Costa, 2007) communities. They may be constructed based on analytical, experimental, or numerical step-response information. They have been applied to a wide range of problems ranging from understanding the effect of control surfaces (Leishman, 1994) to the modeling of gusts (Leishman, 1996), and the suppression of shedding on bridges and buildings (Salvatori and Spinelli, 2006; Costa, 2007). Nonlinear extensions have been developed (Tobak and Chapman, 1985; Prazenica et al., 2007; Balajewicz and Dowell, 2012).

Theodorsen's frequency-domain model is equivalent to Wagner's, using the same model assumptions of an incompressible, inviscid flow with a planar wake. Although Theodorsen's model applies only to sinusoidal input motion, it was suitable for the analysis of flutter instability with the tools available. However, with modern tools, it is possible to construct a state-space realization based on Theodorsen's lift model that is useful for time-domain analysis and feedback control (Brunton and Rowley, 2013). Dinyavari and Friedmann (1986) and Breuker et al. (2008) construct state-space models for Theodorsen's transfer function, but not for the entire lift coefficient. Peters et al. (1995, 2007) and Peters (2008) developed a state-space model based on general potential flow theory and the Glauert expansion of inflow states, and Theodorsen's model is a special case.

Accurate state-space aerodynamic models are especially important when the flight dynamic and aerodynamic time scales are comparable. In this case, modern control techniques such as $\mathcal{H}_\infty$-synthesis can be especially useful for achieving robust performance. Because small, lightweight aircraft have shorter flight dynamic time-scales, small vehicles and bio-flyers at low Reynolds number, $\mathrm{Re} = cU_\infty/\nu$, between 10^2 and 10^5 are particularly interesting; here ν is the kinematic viscosity. However, the classical models are limited by the inviscid assumption that allows for closed-form solution, which makes them less accurate for low Reynolds numbers and at larger angles of attack.