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Observation and evolution of chaos for a cantilever plate in supersonic flow

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ABSTRACT

For a wing-like plate in supersonic flow cantilevered at its root, chaotic motions are studied in this paper. Prior literature has mainly focused on a simply supported plate or the limit cycle oscillations (LCOs) of a cantilever plate. The governing equations are constructed using von Karman plate theory and first-order piston theory. The Rayleigh–Ritz approach is adopted to discretize (and reduce the order of) the partial differential equations of the plate, and the resulting ordinary differential equations (ODEs) are solved numerically by the fourth-order Runge–Kutta (RK4) method. Numerical simulations demonstrate that the evolution of chaos is very complex, and the route to chaos depends on the panel's length-to-width ratio a/b. For $a/b=1$, a period-doubling of periodic motion occurs before transition to chaos. Another route to chaos is via quasi-periodic response directly for $a/b=2$. The most complicated prechaos and postchaos pattern is for $a/b=0.5$, which shows the presence of chaos regions with periodicity windows. Additionally, bifurcation diagrams show that certain features of the aeroelastic system such as quasi-periodic motions may be missed with too few Rayleigh–Ritz modes. Time histories, phase plane portraits, Poincaré maps and frequency spectra are used for identifying periodic and chaotic motions.

1. Introduction

Supersonic panel flutter has been investigated by many researchers. Most of them, however, focus on a clamped or simply supported plate undergoing supersonic flow. For this model, Dowell (1966, 1967) first applied the Galerkin method to study the nonlinear oscillations of a fluttering plate in two and three dimensions, and a stability region defined by the in-plane compressive load and the aerodynamic force was obtained to describe the complicated nonlinear dynamic phenomena. More recently, Epureanu et al. (2004a) explored the dynamics of aeroelastic panels using a finite difference method, a Galerkin approach and a proper orthogonal decomposition (POD) method, in which multiple LCO co-existence and chaotic oscillations were observed. The intrinsic sensitivity of the chaos was applied to detect parametric variations in Epureanu et al. (2004b). Zhou et al. (2012) applied the Galerkin approach to analyze the aeroelastic stability of heated panel with aerodynamic loading on both surfaces. For a three-dimensional plate, a reduced-order model (ROM) based on the POD method along with Galerkin projection has been constructed by Xie et al. (2014a) to solve the nonlinear oscillations, and the

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Nomenclature

a, b	plate length, plate width
a_{ij}, b_{rs}	mode coordinate for inplane displacement u, v
D, E	plate stiffness, Young's modulus
h	plate thickness
I, J	total mode number retained in the x, y directions for inplane displacement u
i, j	mode number in the x, y directions for inplane displacement u
$L = T - U$	Lagrangian
Ma	Mach number
M, N	total mode number retained in the x, y directions for transverse deflection
m, n	mode number in the x, y directions for transverse deflection
Δp	aerodynamic pressure, positive in direction opposite to w
Q	generalized aerodynamic force
q	dynamic pressure, $\rho_\infty V_\infty^2/2$
q_{mn}	mode coordinate for transverse deflection
R, S	total mode number retained in the x, y directions for inplane displacement v
r, s	mode number in the x, y directions for inplane displacement v
T	kinetic energy
t	time
U	total elastic energy
u, v	in-plane displacement in length and width
$u_{i(r)}, v_{j(s)}$	mode in the x, y directions for in-plane displacement $u(v)$
V_∞	flow velocity
w	panel transverse deflection
x, y, z	streamwise, spanwise, normal coordinates
β	$(Ma^2-1)^{1/2}$, compressibility correction factor
λ	$2qa^3/\beta D$, nondimensional dynamic pressure
μ	$\rho a/\rho_m h$, nondimensional fluid/structure mass ratio
ν	the Poisson ratio
ξ, η	$x/a, y/b$
ρ, ρ_m	air density, plate density
τ	$t(D/\rho_m h a^4)^{1/2}$, nondimensional time
ϕ_m, ψ_n	mode in the x, y directions for transverse deflection w
$()', \dot{()}$	$d()/d\xi$ or $d()/d\eta$, $d()/d\tau$

computational cost was significantly reduced, especially for panels with large length-to-width ratios. In Xie et al. (2014a), it was shown that chaotic response is crucial for constructing ROM, which is a primary motivation for the present work.

For chaotic oscillations, Dowell (1982) observed the chaotic self-excited oscillations in a simply supported fluttering buckled plate system. Li and Yang (2014) studied the stability and chaos of a cantilever plate subjected to subsonic flow. Thus, chaos for a simply supported plate in supersonic flow (Epureanu et al., 2004a; Dowell, 1982) and a cantilever plate in subsonic flow (Li and Yang, 2014) has been observed. However, whether or not it occurs for the cantilever plate in supersonic flow has not been determined previously. In the present study, we will show that the complex motions do arise for the cantilever plate subjected to supersonic flow.

A transition from periodic to chaotic motions may occur through parameter changes (Moon, 1987). One should try to vary one or more of the control parameters in the system to observe the evolution for chaotic oscillations. Dynamic pressure λ will be the primary control parameter in this paper. A bifurcation diagram for observing the prechaotic or postchaotic behaviors is mapped out via sweeping the parameter λ, and plotting local amplitude extrema of transverse deflection. The boundary between periodic and chaotic motions can be seen directly in the bifurcation diagrams.

In this study, time histories, phase portraits, Poincaré maps and frequency spectra are the main descriptors to observe the evolution of chaos. In a Poincaré map, a set of finite K fixed points indicates a K-period motion; a quasi-period motion where two or more incommensurate periodic motions are present is shown as a closed curve; and a cloud of discrete points implies a chaotic motion. Essentially, a Poincaré map provides a means to visualize the strange attractor when the motion is chaotic. Frequency spectra via Fast Fourier Transform (FFT) of time response is one of the quantitative measures. When the motion is periodic or quasi-periodic, frequency spectra show a single or a set of narrow spikes. On the other hand, a continuous and broadband spectrum is a crucial clue of possible chaotic motions. Readers are referred to Moon (1987) for more details.

In this paper, evolution of chaos for a cantilever plate in supersonic flow is explored, as an extension work to Ye and Dowell (1991) and Hopkins (1994). The structure of this paper is organized as follows. Section 2 applies the classical Rayleigh–Ritz method to solve the aeroelastic equations. In Section 3.1.1, a convergence study is performed based on bifurcation diagrams that give careful suggestions on mode truncation. Meanwhile, a route to chaos for panel with length-to-width ratio $a/b = 1$ is presented. Section 3.1.2 proves the symmetry of the aeroelastic system mathematically, and duality of responses is observed from numerical examples. Evolution of chaos for panels of $a/b = 0.5$ and $a/b = 2$ is studied, and distinct routes to chaos are obtained in Sections 3.2 and 3.3, respectively. Deflection shapes of LCO and chaos for different panels are investigated in Section 3.4. Finally, the main conclusions are drawn in Section 4.

2. Theoretical analysis

In this section, the Rayleigh–Ritz method is employed to explore the nonlinear dynamics of the cantilever plate shown in Fig. 1. The model studied here is a two-dimensional plate constrained along the root edge with supersonic flow over the