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The identification of coherent structures using proper orthogonal decomposition and dynamic mode decomposition

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A B S T R A C T

A comprehensive comparison was conducted on the identification of coherent structures in fluid flow using Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD). The influences that multi-dominant structures and high-order harmonics had on the decomposed modes were taken into extensive consideration. To this end, a series of fabricated patterns was constructed for the benchmark testing to simulate multi-dominant convective structures superimposed in a stationary field. The comparison showed that the temporal DMD method could clearly separate each structure in the spatial and spectral senses, while the POD mode corresponding to the desired structure is contaminated by the other uncorrelated structures. Subsequently, two case studies of the real wake flows, which were determined from high-repetition TR-PIV measurements, were employed to demonstrate the discrepancies of the POD and DMD algorithms in extracting coherent structures. For the wake flow behind a single cylinder at $Re_D=8000$, the temporal DMD algorithm accurately determined the frequency, wavelength and convection speed of the Karman-like vortex street and its higher-order harmonics. However, although the first two POD modes are closely related to the Karman-like vortex street, the higher POD modes embedded as larger structures are obscure in the physical significance. Finally, the wake flow behind two side-by-side cylinders of different diameters at $Re_D=1000$ based on the diameter of the small cylinder was measured; two configurations with different gaps were chosen for comparison, i.e., $G/D=0.5$ and 2.0. For the wake flow at $G/D=0.5$, the POD and DMD algorithms determined the major features of the single-dominant structure. For the wake flow at $G/D=2.0$, the first and second temporal DMD modes effectively and independently extracted the Karman-like vortex structures behind the large and small cylinders, respectively. Meanwhile, although the first and second pairs of POD modes generally captured these two convecting structures, respectively, there was obvious existence of the undesirable contamination of the POD mode, as reflected in the interaction between the desired and uncorrelated structures.

1. Introduction

Fluid flows such as wakes, jets, and separated flows are largely characterized by coherent structures with different spatial and temporal scales and the common existence of multi-dominant structures in fluid flows makes identifying the spatio-temporal

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Nomenclature

A	matrix comprised of mode coefficients $a_i(t)$
B	system matrix
C	companion matrix
$\tilde{C}$	full companion matrix
D	diameter of small cylinder [m]
G	gap between two cylinders [m]
G_{aa}	power spectra of coefficient $a(t)$
M	finite number of POD modes
R	temporal correlation matrix
Re_D	Reynolds number based on diameter of small cylinder
S	infinite Vandermonde matrix
$\tilde{S}$	finite Vandermonde matrix
K	Koopman operator
U	discrete velocity matrix
U_0	free stream velocity [m/s]
$a_i(t)$	mode coefficients of POD
a_n	constant
b_n	constant
c_i	$c=(c_1,\dots,c_{N-1},c_N)$
d	diameter of glass beads [m]
d_n	size factor of fabricated structure
e	$e=(0,\dots,0,1)^T$
f	frequency [Hz]
$f()$	a map comparing a manifold to itself
g	vector-valued observable
q	fabricated pattern
q_n	component of the fabricated pattern
r	residual vector
T	period of the shedding structures
Δx	distance between two adjacent points along the streamwise direction
u	instantaneous velocity [m s^{-1}]
$\overline{u}$	time-mean velocity [m s^{-1}]
u'	fluctuation velocity [m s^{-1}]
v_j	Koopman mode

Greek symbols

$\alpha_n(t)$	growth rate factor
β_n	wavelength factor
γ_n	velocity factor
δ	thickness of turbulent boundary layer
Λ	diagonal matrix with eigenvalues
λ_x	characteristic wavelength along the streamwise direction
λ_i	eigenvalues of Koopman operator
μ	eigenvalues mapping onto the complex plane
ρ	density of the glass beads [kg m^{-3}]
σ	growth rate of DMD modes
$\sigma_k(x)$	spatial eigenfunctions
Φ	dynamic modes
$\|\|\Phi\|\|$	energy norm of dynamic mode
φ_i	Koopman eigenfunctions
ω	phase velocity of DMD modes

Abbreviations

DMD	Dynamic Mode Decomposition
DNS	Direct Numerical Simulation
LES	Large Eddy Simulation
PIV	Particle Image Velocimetry
POD	Proper Orthogonal Decomposition
RAID	Redundant Arrays of Inexpensive Disks
SSD	Solid State Disk
TR-PIV	Time-resolved Particle Image Velocimetry
WAG	Window Average Gradient

features of each coherent structure challenging. Such identification, however, plays a significant role in study of various physical processes, e.g., heat and mass transfer, and flow noise. Recent decades have seen rapid developments in remarkable applications of Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS), along with the state-of-the-art Time-resolved Particle Image Velocimetry (TR-PIV) – all of which have enabled the accurate capture of instantaneously varying flows at reasonable spatial and temporal resolutions. It is highly desirable to extract the salient coherent structures from a large quantity of numerical and experimental information.

In the last two decades, many techniques have been proposed to identify the coherent structures, e.g., vorticity concentration, critical points, Window Average Gradient (WAG) detection and Q-criterion. By plotting vorticity contour maps, Hussain and Hayakawa (1987) characterized large-scale organized structures in the turbulent plane wake of a circular cylinder; the spatially phase-correlated vorticity was employed to examine the dynamics of coherent structures in turbulent shear flow (Hayakawa and Hussain, 1989) and cylinder wake (Zhou and Antonia, 1993). Using the Q-criterion, the kinematical and dynamical properties of the flow, for example kinetic energy, Reynolds stress, were made clear by describing the flows in terms of individual events or streamline patterns (Hunt et al., 1988). Bisset et al. (1990) detected the organized motion in a cylinder wake by calculating the WAG value. Zhou and Antonia (1994) applied the critical point method in analysis of a turbulent near wake, demonstrating that the spanwise vortices provide the dominant contribution to the Reynolds shear stress. The above-mentioned methods have been extensively employed in fluid-mechanics community to determine characteristics of the vortices buried in turbulent flows. However, such methods would obscure the underlying complexity of fluid flow superimposed with multi-dominant coherent structures. Among all of the data-driven algorithms that have been used to determine the spatio-temporal features of multi-scale coherent structures, Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD) have enjoyed widespread use. A comprehensive understanding of the roles of POD and DMD algorithms in the identification of multi-dominant coherent structures in spatial and spectral domains is of essential significance.

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