Journal of Fluids and Structures ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Improving vortex models via optimal control theory

Maziar S. Hemati*, Jeff D. Eldredge, Jason L. Speyer

Department of Mechanical and Aerospace Engineering, University of California, Los Angeles, CA 90095, USA

ARTICLE INFO

Article history:
Received 27 June 2013
Accepted 13 April 2014

Keywords:
Model optimization
Optimal control
Point vortex
Unsteady aerodynamics

ABSTRACT

Low-order inviscid point vortex models have demonstrated success in capturing the qualitative behavior of aerodynamic forces resulting from unsteady lifting surface maneuvers. However, the quantitative agreement is often lacking for separated flows as a result of the ambiguity in the edge conditions in this fundamentally unsteady process. In this work, we develop a model reduction framework in which such models can be systematically improved with empirical results. We consider the low-order impulse matching vortex model in which, in its original form, Kutta conditions are applied at both edges to determine the strengths of single point vortices shed from each edge. Here, we relax the Kutta condition imposed at the plate's edges and instead seek the time rate of change of the vortex strengths that minimize the discrepancy between the model-predicted and high-fidelity simulation force histories, while the vortex positions adhere to the dynamics of the low-order model. A constrained minimization problem is constructed within an optimal control framework and solved by means of variational principles. The optimization approach is demonstrated on several unsteady wing maneuvers, including pitch-up and impulsive translation at a fixed angle of attack. Additionally, a stitching technique is introduced for extending the time interval over which the model is optimized.

1. Introduction

An improved understanding of the unsteady effects associated with high angle of attack maneuvers has great potential for advancing the capabilities of modern flight systems. For example, by exploiting the leading edge vortex (LEV), an aircraft may be able to realize increased lift or maneuverability by means of delayed stall. Despite recent advances in the field of unsteady aerodynamics, however, low-order models for predicting the aerodynamic forces and moments are still inadequate for designing reliable flight control systems for successfully conducting agile maneuvers. A primary contributor to this shortcoming is a lack of low-dimensional aerodynamic models capable of reliably predicting forces and moments over a wide-range of kinematics.

Low-order modeling of unsteady aerodynamics initially started with the work of Wagner (1925) and Theodorsen (1935). These early studies established a precedent for analyzing such problems by decomposing the forces and moments on the wing into contributions from circulatory (i.e., vortex induced) and non-circulatory (i.e., inertial reaction, or added mass) effects. Many researchers have taken similar phenomenological approaches to modeling. For example, an assortment of

* Corresponding author. Present address: Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA. Tel.: +1 408 761 5001.
E-mail address: mhemati@princeton.edu (M.S. Hemati).

http://dx.doi.org/10.1016/j.jfluidstructs.2014.04.004
0889-9746/

Please cite this article as: Hemati, M.S., et al., Improving vortex models via optimal control theory. Journal of Fluids and Structures (2014), http://dx.doi.org/10.1016/j.jfluidstructs.2014.04.004

Nomenclature

a	semi-chord of plate
c	chord of plate
C_d	drag coefficient
C_l	lift coefficient
F_x, F_y	components of force
$\mathcal{H}$	optimization Hamiltonian
J	objective function
K	dimensionless pitch rate, $\dot{\alpha}_0 c/(2U)$
$\mathbf{p}$	optimization costate vector
Re	Reynolds number
$\mathbf{u}$	optimization input vector
U	speed of translation
$\mathbf{x}$	optimization state vector
$\mathbf{x}_o$	optimization initial state vector
z	complex coordinate, $x+iy$, in physical plane
$\tilde{z}$	plate-fixed coordinates
z_c	plate centroid
z_ν	position of vortex ν in physical plane
$z_{\nu 0}$	position of the releasing edge of vortex ν
α	angle of attack
$\dot{\alpha}_0$	nominal dimensional pitch rate
Γ_ν	strength of vortex ν
ϵ	gradient descent threshold stopping criterion
ζ	complex coordinate, $\xi+i\eta$, in circle plane
ζ_ν	position of vortex ν in circle plane
θ	angular coordinate in circle plane
κ_i	gradient descent step size for parameter i
ν	fluid kinematic viscosity
ρ	fluid density

vortex models have been developed to account for the shed vorticity through a multitude of simple vortex representations, such as vortex sheets (Wagner, 1925; Theodorsen, 1935; Garrick, 1937; von Kármán and Sears, 1938; Krasny, 1991; Nitsche and Krasny, 1994; Jones, 2003; Pullin and Wang, 2004; Shukla and Eldredge, 2007; Alben and Shelley, 2008), continuous sequences of point vortices (Katz and Weihs, 1978; Jones and Platzer, 2000; Ansari et al., 2006; Ramesh et al., 2013; Xia and Mohseni, 2013), or finite sets of point vortices with evolving strengths (Brown and Michael, 1954; Graham, 1980; Cortelezzi and Leonard, 1993; Michelin and Llewelyn Smith, 2009; Wang and Eldredge, 2013).

Many of these classical potential flow models perform well for low angles of attack, but fail to provide reliable force predictions when the angle of attack is increased to the point that the LEV plays a significant role. The impulse matching model, a low-order variable-strength vortex model, has recently been developed to address this issue (Wang and Eldredge, 2013). The model, which makes use of a Kutta condition at the wing's leading and trailing edges in order to determine the strengths of evolving point vortices, provides reasonable force predictions in many cases; however, the model still remains inadequate for aerodynamic control and estimation. This is not entirely surprising, since the Kutta conditions are primarily used due to the lack of a better model. In reality viscous and curvature effects play a significant role, especially at the leading edge, thus making the Kutta condition an ill-suited model for maneuvers with LEV development.

It may be possible to circumvent these modeling deficiencies by synthesizing empirical data to construct a more reliable alternative. For example, several researchers have successfully improved upon template models by tailoring them based on relevant empirical data. In one such model, experimental force data is used to determine the strengths and positions of a set of stationary point vortices (Pitt Ford and Babinsky, 2013). Similarly, Wong et al. (2013) implement a vortex-feeding model to estimate the time-varying circulation of a wing from time-resolved measurements of the leading edge shear layer velocity profile. Another example, from Brunton and Rowley (2011, 2013), makes use of an empirically determined Theodorsen function to improve upon Theodorsen's classical lift model. Ramesh et al. (2012) augment a point vortex model with an empirically determined leading edge suction parameter to govern vortex shedding from the leading edge of an airfoil. Additionally, the method of indicial responses, considered in Leishman and Beddoes (1989) and more recently in Taha et al. (2013), enables the construction of empirically guided models capable of reasonable force and moment predictions.

In the present work, we develop a general framework for exploiting empirical data to improve upon existing vortex models. Specifically, we relax the Kutta conditions imposed at the leading and trailing edges of the impulse matching vortex model (Wang and Eldredge, 2013) and instead determine the vortex strengths by minimizing the difference between some model predicted and high-fidelity simulation metric (here, force). The minimization is constrained to ensure that vortices move according to specified dynamical equations. Although the resulting model is not autonomous, it provides insight into the deficiencies of the original model and may guide future model development.

We summarize the impulse matching model in Section 2, which will serve as the baseline vortex model for the minimization procedure. The minimization problem is formulated in Section 3, which also details the method of solution and the viscous vortex particle method used in acquiring "truth" data for the optimization procedure. In Section 4 we apply the optimization procedure to two canonical wing maneuvers: (1) pitch-up and (2) impulsive translation at a fixed angle of attack. Finally, in Section 5, we discuss some of the remaining challenges in vortex model optimization.

2. Vortex model overview

In the present section, we briefly introduce the impulse matching model for predicting the aerodynamic forces of a pitching and/or translating airfoil. A more detailed development can be found in Wang and Eldredge (2013).

Please cite this article as: Hemati, M.S., et al., Improving vortex models via optimal control theory. Journal of Fluids and Structures (2014), http://dx.doi.org/10.1016/j.jfluidstructs.2014.04.004