Journal of Fluids and Structures 49 (2014) 242–262

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Finite-depth effects on solitary waves in a floating ice sheet

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A R T I C L E I N F O

Article history:
Received 11 April 2013
Accepted 28 April 2014
Available online 28 May 2014

Keywords:
Finite depth
Flexural-gravity waves
Hamiltonian theory
Solitary waves
Water waves

A B S T R A C T

A theoretical and numerical study of two-dimensional nonlinear flexural-gravity waves propagating at the surface of an ideal fluid of finite depth, covered by a thin ice sheet, is presented. The ice-sheet model is based on the special Cosserat theory of hyperelastic shells satisfying Kirchhoff's hypothesis, which yields a conservative and nonlinear expression for the bending force. From a Hamiltonian reformulation of the governing equations, two weakly nonlinear wave models are derived: a 5th-order Korteweg–de Vries equation in the long-wave regime and a cubic nonlinear Schrödinger equation in the modulational regime. Solitary wave solutions of these models and their stability are analysed. In particular, there is a critical depth below which the nonlinear Schrödinger equation is of focusing type and thus admits stable soliton solutions. These weakly nonlinear results are validated by comparison with direct numerical simulations of the full governing equations. It is observed numerically that small- to large-amplitude solitary waves of depression are stable. Overturning waves of depression are also found for low wave speeds and sufficiently large depth. However, solitary waves of elevation seem to be unstable in all cases.

1. Introduction

In recent years, there has been renewed interest in the study of flexural-gravity (or hydroelastic) waves at the surface of a fluid covered by a thin elastic sheet, with applications to ocean waves interacting with sea ice in polar regions (Korobkin et al., 2011). A number of experiments have been performed with moving loads on ice, e.g. at McMurdo Sound, Antarctica, in deep water (Squire et al., 1996) and on Lake Saroma, Japan, in shallow water (Takizawa, 1985).

A theoretical challenge in this problem is to model the ice deformations subject to water wave motions, and thus a number of models have been proposed. The linear Euler–Bernoulli model for the ice sheet, combined with potential flow, has been widely used for small-amplitude water waves and ice deflections (Squire et al., 1996; Meylan and Sturova, 2009; Montiel et al., 2012; Mohapatra et al., 2013). However, reports of intense-in-ice events have highlighted limitations of linear theory (Marko, 2003) and, with the perspective of rougher sea conditions due to global warming (Squire, 2011), nonlinear theory has drawn increasing attention.

Nonlinear models based on Kirchhoff–Love plate theory have been adopted by a number of investigators (Forbes, 1986; Hegarty and Squire, 2008; Milewski et al., 2011). Recently, Plotnikov and Toland (2011) proposed a nonlinear formulation based on the special Cosserat theory for hyperelastic shells, which has the advantage of conserving elastic energy unlike the

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http://dx.doi.org/10.1016/j.jfluidstructs.2014.04.015

Kirchhoff–Love model. In this Cosserat framework, Milewski et al. (2011) performed a weakly nonlinear analysis of two-dimensional hydroelastic solitary waves on deep water using the multiple-scale method, while Guyenne and Părău (2012) examined the same problem both analytically and numerically through a Hamiltonian reformulation of the governing equations. In particular, both studies derived a defocusing nonlinear Schrödinger (NLS) equation implying the non-existence of small-amplitude solitary waves in the deep-water case.

The present paper extends the work of Guyenne and Părău (2012) to finite depth. Unlike infinite depth, there are two critical wave speeds: the minimum phase speed $c_{\min}$ and the long-wave (or shallow-water) limit c_0. In the past, this problem has been investigated through weakly nonlinear modelling (Haragus-Courcelle and Ilichev, 1998; Părău and Dias, 2002; Xia and Shen, 2002) and direct numerical simulations (Bonnefoy et al., 2009; Părău and Vanden-Broeck, 2011; Vanden-Broeck and Părău, 2011) using either the linear Euler–Bernoulli or nonlinear Kirchhoff–Love model for the ice sheet, combined with nonlinear potential flow. In particular, Xia and Shen (2002) derived a 5th-order Korteweg–de Vries (KdV) equation for two-dimensional hydroelastic waves on shallow water in the Euler–Bernoulli case, and Haragus-Courcelle and Ilichev (1998) derived a three-dimensional generalisation of the 5th-order KdV equation in a similar setting. Părău and Dias (2002) used Kirchhoff–Love plate theory and derived a stationary NLS equation for wave speeds c near $c_{\min}$. They found that this equation becomes focusing in shallow water and thus admits solitary wave solutions. More recently, Milewski and Wang (2013) proposed a Benney–Roskes–Davey–Stewartson system for three-dimensional weakly nonlinear hydroelastic waves on finite depth, based on the formulation of Plotnikov and Toland (2011).

Following Guyenne and Părău (2012), our starting point is the Hamiltonian formulation of the hydroelastic problem. The Dirichlet–Neumann operator is introduced to reduce the original Laplace problem to a lower-dimensional system involving quantities evaluated at the fluid–ice interface only. We restrict our attention to free solitary waves in two dimensions. Our weakly nonlinear analysis includes the derivation of a 5th-order KdV equation for c near c_0 and of a cubic NLS equation for c near $c_{\min}$, using the Hamiltonian perturbation approach of Craig et al. (2005a, 2010). We find that this NLS equation is of focusing type below a critical depth, similar to results of Părău and Dias (2002), and thus admits stable soliton solutions. Larger-amplitude waves for $c<c_{\min}$ are computed by solving the full steady equations with a boundary-integral method. Interestingly, their profiles can be well approximated by NLS solutions, despite being outside the regime of validity of the NLS equation. Both solitary waves of depression and elevation are found, including overturning waves of depression for c much lower than $c_{\min}$ and sufficiently large depth. For very shallow water, depression waves cannot reach a sufficiently high amplitude to overturn as they are bounded by the fluid depth.

The stability of these solitary waves is also investigated by performing direct numerical simulations in time with a high-order spectral method. Thanks to its analyticity properties, the Dirichlet–Neumann operator has a convergent Taylor series expansion in which each term can be determined recursively. This series expansion combined with the fast Fourier transform leads to an efficient and accurate numerical scheme for solving the full Hamiltonian equations. For $c<c_{\min}$, the stability of solitary waves of depression is confirmed by our time-dependent computations, while solitary waves of elevation seem to be unstable. For $c>c_0$, we observe solitary waves of elevation which compare well with generalised solitary wave solutions of our 5th-order KdV equation. Such waves however are inherently unstable because they continuously emit radiation and thus decay in time.

The remainder of the paper is organised as follows. Section 2 presents the mathematical formulation of the hydroelastic problem. The Dirichlet–Neumann operator is introduced and the Hamiltonian equations of motion are established. From this Hamiltonian formulation, the weakly nonlinear wave models are derived and analysed in Section 3. Section 4 describes the numerical methods to solve the full nonlinear problem for steady and unsteady waves. Numerical results are shown and discussed in Section 5. Finally, concluding remarks are given in Section 6.

2. Formulation

2.1. Equations of motion

We consider a two-dimensional fluid of uniform finite depth h beneath a continuous thin ice sheet. The fluid is assumed to be incompressible and inviscid, and the flow to be irrotational. The ice sheet is modelled using the special Cosserat theory of hyperelastic shells in Cartesian coordinates (Plotnikov and Toland, 2011), with the x-axis being the bottom of the ice sheet at rest and the y-axis directed vertically upwards. The vertical deformation of the ice is denoted by $y=\eta(x,t)$. The fluid velocity potential $\Phi(x,y,t)$ satisfies the Laplace equation

$$\nabla^2\Phi=0, \quad \text{for } x\in\mathbb{R}, \quad -h<y<\eta(x,t). \tag{1}$$

The nonlinear boundary conditions at $y=\eta(x,t)$ are the kinematic condition

$$\eta_t+\Phi_x\eta_x=\Phi_y, \tag{2}$$

and the dynamic condition

$$\Phi_t+\frac{1}{2}\left(\Phi_x^2+\Phi_y^2\right)+g\eta+P(x,t)+\frac{\mathcal{D}}{\rho}\left(\kappa_{ss}+\frac{1}{2}\kappa^3\right)=0, \tag{3}$$