ARTICLE IN PRESS

Journal of Fluids and Structures ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Nonlinear vibrations of cantilevered circular cylindrical shells in contact with quiescent fluid

M. Paak [a,b,*], M.P. Païdoussis [a], A.K. Misra [a]

[a] Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, Québec, Canada H3A 0C3
[b] Fluid-structure interaction Lab., McGill University, 817 Sherbrooke Street West, Montreal, Québec, Canada H3A 0C3

ARTICLE INFO

Article history:
Received 29 July 2013
Accepted 28 April 2014

Keywords:
Cylindrical cantilevered shells
Nonlinear vibration
Geometric imperfections
Extended Hamilton's principle
Quasiperiodic and chaotic oscillations
Potential flow theory

ABSTRACT

Large-amplitude vibrations of liquid-filled cantilevered (clamped–free) circular cylindrical tanks are studied theoretically for the first time. The influence of liquid height and initial geometric imperfections is investigated in detail. The tank motions are described by a nonlinear model based on Flügge's shell theory, and the liquid motions are modelled by means of linearized potential flow theory. Equations of motion are obtained using the extended Hamilton's principle and are discretized by expanding the solution with trigonometric functions in the circumferential direction and the cantilevered beam eigenfunctions in the axial direction. The geometric boundary conditions are satisfied exactly, while the natural ones are satisfied in an energy minimization sense. The system is integrated numerically by employing the appropriate modal composition of the solution to guarantee convergence. Results are presented in the form of frequency–response curves in the neighbourhood of the lowest natural frequency. It is found that the response may be of softening or hardening type, depending on the liquid height and the imperfection parameters.

1. Introduction

Shells in contact with fluid are found in many science and engineering applications. Of particular importance are shells containing dense fluid because of their broad applications such as containers, reservoirs, and silos. Although these structures are designed to be subjected to small motions under typical loads, there are instances, *e.g.*, under seismic loading, in which the system may undergo large motions. Therefore, it is necessary to study the large-amplitude (nonlinear) vibrational behaviour of shells containing dense (heavy) fluid, in order to improve design criteria and avoid failures.

Amabili and Païdoussis (2003) conducted a comprehensive review of the literature on the nonlinear vibrations of circular cylindrical shells. This review shows that most of the work on this subject deals with simply supported shells, whereas the nonlinear vibrations of *cantilevered* shells, despite their appearance in many applications, have not been studied much. Some of the key contributions, the results of which are relevant to the present study, are discussed in the following.

The study of nonlinear vibrations of shells *in vacuo* is important and reveals the basic characteristics of their nonlinear behaviour. Prominent early work in this area may be attributed to Evensen (1963, 1967, 1968). In these studies, Donnell's nonlinear shallow-shell theory was used, and the shell was circular cylindrical with simply supported ends. It was found that cylindrical shells

* Corresponding author. Present address: Aerodynamics Lab., École polytechnique de Montréal, Canada.
E-mail address: mehdi.paak@mail.mcgill.ca (M. Paak).

http://dx.doi.org/10.1016/j.jfluidstructs.2014.04.017
0889-9746/© 2014 Elsevier Ltd. All rights reserved.

often exhibit softening-type nonlinear behaviour. Using a different approach, Dowell and Ventres (1968) noted that shells with large length-to-radius ratio behave like a ring and display softening nonlinearity, whereas for shells with small length-to-radius ratio hardening nonlinearities should be expected. Ginsberg (1973, 1974) studied the nonlinear free and forced vibrations of a simply supported shell using Flügge's shell theory, which is more accurate than Donnell's shallow shell theory. Depending on the system parameters, both hardening- and softening-type behaviour was observed. Chen and Babcock (1975) used a perturbation technique in conjunction with the nonlinear Donnell equations to investigate nonlinear vibrations of a simply supported shell; they found that the nonlinearity is of a softening type. Their experimental and theoretical results agree well. The nonlinear vibrations of simply supported shells *in vacuo* were also examined by Amabili et al. (1998) as part of their study on the nonlinear vibrations of fluid-filled shells; the results are in good agreement with the experimental results of Olson (1965) and also with the results of Evensen (1967) and Ganapathi and Varadan (1996).

The literature on the large-amplitude vibrations of shells containing dense fluid is less extensive. Gonçalves and Batista (1988) studied free and forced nonlinear vibrations of fluid-filled simply supported shells using Sander's nonlinear shell theory; the fluid is assumed to be inviscid and is modelled by potential flow theory. It is found that the shell response is more softening in the presence of fluid than in vacuum.

Amabili et al. (1998) investigated the nonlinear free and forced vibrations of simply supported circular cylindrical shells in contact with quiescent dense fluid. Donnell's nonlinear shallow shell theory is employed. Equations are solved both numerically and using a perturbation analysis. Strong softening nonlinearity is observed for the fluid-filled shell, and it is concluded that the fluid augments the nonlinearity of the system. Amabili et al. (2000) studied the nonlinear behaviour of simply supported shells *in vacuo* and filled with quiescent fluid under a radial harmonic excitation, using a more refined model. They also performed experiments on a steel shell filled with water. The theoretical and experimental results are in very good agreement; they confirm their previous finding that, the fluid-filled shell exhibits stronger softening nonlinearity than the shell *in vacuo*. The same conclusion is drawn by Karagiozis et al. (2005) for fluid-filled clamped circular cylindrical shells. They constructed two models: one based on Donnell's shallow shell theory and another based on an energy approach. Theoretical results from both models are in good agreement with the experimental results of Chiba (1993a) for partially filled clamped shells.

There are a number of studies addressing the influence of geometric imperfections on the nonlinear vibrations of simply supported shells. Watawala and Nash (1983) used Donnell's shallow shell theory and considered the fluid free surface condition. They found that imperfections with the same circumferential wavenumber as the mode being investigated have a softening effect. In a systematic analysis, Amabili (2003a,b) found that (i) axisymmetric imperfections do not split the double frequency associated with the driven and companion modes, (ii) oval imperfections have little effect on the natural frequency of modes with several circumferential waves, they too do not split the double frequencies, (iii) imperfections with twice the number of circumferential waves of the mode being analysed have a greater effect than with the same number of circumferential waves; however, in both cases these imperfections split the double frequencies.

Large-amplitude vibrations of supported-end shells in contact with flowing fluid have been studied by Lakis and Laveau (1991), Selmane and Lakis (1997), Amabili et al. (1999) and Karagiozis et al. (2008, 2010). A complete account of the nonlinear dynamics and stability of shells in contact with flowing fluid is given by Païdoussis (2004).

All the aforementioned studies are concerned with the nonlinear dynamics of supported-end shells; the nonlinear behaviour of cantilevered (clamped–free) shells, however, has been addressed by very few only. Chiba (1993b) studied experimentally the large-amplitude vibrations of cantilevered circular cylindrical shells *in vacuo*. He tested two shells of different lengths, which were made of polyester sheet and were harmonically excited in the radial direction. He found that almost in all cases the shell response is of softening type, and the shorter shell displays stronger softening behaviour. Also, among the modes with the same axial wavenumber, the weakest degree of nonlinearity is associated with the mode with the lowest natural frequency.

It was only recently that Kurylov and Amabili (2011) studied theoretically the nonlinear vibrations of cantilevered shells *in vacuo* for the first time, using Sanders nonlinear shell theory. The solution is expanded with trigonometric functions in the circumferential directions and with Chebyshev polynomials in the axial direction. The system dimension is reduced by replacing axisymmetric modes with “artificial” ones which have the axial shape of asymmetric modes. The shell shows very weak softening-type behaviour. The effect of imperfections is also studied; it is found that axisymmetric imperfections make the system response hardening, while oval imperfections make it slightly softening.

The only research exploring the nonlinear vibrations of cantilevered shells containing fluid is the experimental study by Chiba (1993c). He used polyester shells of two different lengths, partially filled with water to different levels. It was observed that the nonlinearity depends on the system parameters, such as the shell geometry and liquid level. The partially filled shell has softening response, in general; however, hardening or hardening–softening nonlinearites were also seen for some vibration modes. For each liquid level, among the modes with the same axial wavenumber the weakest degree of softening nonlinearity belongs to the mode with the minimum natural frequency. In general, shells filled to a lower level have a stronger softening nonlinearity.

The large-amplitude vibrations of cantilevered shells containing flowing fluid has recently been studied theoretically by Paak et al. (2013, 2014) for the first time. In this paper, however, we study theoretically the nonlinear vibrations of cantilevered shell containing dense *quiescent* fluid. A nonlinear model is developed based on Flügge's shell theory; the fluid motions are described by linearized potential flow theory. The response of the system to radial harmonic excitation in the spectral neighbourhood of the lowest natural frequencies is calculated numerically. The effects of geometric imperfections and liquid height on the dynamics of the system are investigated.